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The Elastic Stability of Sandwich Plates

-by-

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SUMMARY

This paper treats the elastic stability of supported rectangular plates of sandwich construction with isotropic and aeolotropic fillings under compression and shear loading. Formulae are developed for critical stresses for flat and curved panels in compression and flat panels in shear for the buckling of the whole panel, also for the wrinkling or local failure of the faces of flat panels in compression. A method is indicated for calculating the critical load of a cylinder in pure bending.

It is established that for a wide range of conditions the critical stress for panels buckling in compression is independent of the form of the filling providing it is symmetrical about the normal; of the elastic constants of the filling only the transverse shear is of concern. As a result a simple extension of the equivalent plate theory of greatly improved accuracy is developed enabling the use of equations treating the plate as a whole.

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NOTE: This paper was presented as a thesis for the Diploma of the College of Aeronautics, June 1948.

BHF

CONTENTS

1. SUMMARY OF RESULTS

An isotropic and two forms of aeolotropic filling are considered. Of the latter both are symmetrical about the normal to the panel*, but one has stiffnesses only in normal planes and the other has comparable stiffnesses in all planes: the first of these will be referred to as a honeycomb filling, as a fabricated paper honeycomb filling is a good example of the material in mind, and the other simply as aeolotropic.

Failure may be either a buckling of the panel as a whole or a short wave buckling or wrinkling of the faces. The latter has only been investigated for simply supported flat panels. The critical stress for this type of failure is not dependent on the dimensions of the panel other than the ratio of the skin thickness to the overall thickness; from this and from dimensional arguments there is reason to believe that the wrinkling stress is independent of the edge conditions and of curvature and that a panel in shear will wrinkle when the compression field equals the critical compressive wrinkling stress.

Sections 4 and 5 of this paper derive by a fairly rigorous analysis formulae for the critical stresses for buckling, as opposed to wrinkling, of simply supported panels in compression which divide into three orders of approximation. The first, which in general is quite inaccurate, is the result of ignoring shear deformation and gives the so-called equivalent plate stress; this, of course, is the same for all forms of filling. The second, which is also independent of the form of filling, provides in effect a correction factor to the equivalent plate stress that is a function of a parameter, denoted by the symbol η, which is a measure of the transverse shear stiffness of the filling in terms of the strength of the whole panel in compression. The third approximation is a more complicated expression giving the failing stress for panels that are weak in shear.

This common second approximation is important because it is accurate over a wide range of conditions and because it can also be deduced by a simple allowance for the shear deformation of the filling. Though the latter is a natural and obvious extension of engineer's bending theory, the rather tedious analysis of sections 4 and 5 of this Report remains necessary because it establishes the accuracy and range of validity of the approximation. The way is then cleared for applying this simple concept of the behaviour of the filling with a fair hope of comparable accuracy to such problems as clamped edge conditions and shear loading which are difficult by more rigorous methods.

/ The smaller ....

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* From recent tests with Dufaylite this is not strictly true for honeycomb material. The transverse shear modulus measured along the direction of a diagonal of a hexagonal cell differs from that measured at right angles.

The smaller the parameter η the more marked is the effect of shear deformation with a consequent decrease in both failing stress and buckling wave length. This applies in all circumstances of long wave buckling.

The problem of curved panels in compression is treated by an approximate correction of the flat panel solution and also as a check by the method of simple shear allowance. No investigation is made of the stability of the buckled state, so that the apparent gain in stiffness may be illusory.

The critical shear stresses for nearly square and for long narrow panels are calculated by energy methods. Both solutions are approximate particularly the latter. Shear critical stresses appear to fall more rapidly with the filling shear stiffness than the corresponding compressive stresses, and it is difficult to estimate the lower limit of the parameter η for the validity of the formulae.

The formulae derived can only be a guide to the failing stress owing to the simplifying assumptions particularly with regard to edge conditions. A general verification of results is of course necessary but especially of the more approximate formulae such as those for curved panels and panels in shear. Among the many other aspects are two which may be of overriding importance, the effect of non-homogeneous fillings and the effect of initial waviness of the skin. The use of large mesh honeycomb material is clearly another source of local failure of the skin. Initial waviness will lead to additional bending stresses in the faces that may cause their premature yielding and impose strains that may rupture the adhesive at the interfaces. The implications of initial waviness have been discussed at length by Howard (ref. 1) and very similar results to his can be obtained by using the formulae derived in this paper.

/ 2. NOTATION ....

2. NOTATION

A, C, L, N, F	Elastic constants of filling (equation 1)
E, σ	Young's modulus & Poisson ratio of faces.
a, b	Panel length and width
2h, t	Filling and face thickness
R	Radius of curvature of panel
f or q	Compressive or shear stress due to applied load
f_c or q_c	Critical compressive or shear stress
X_x, Y_z etc.	Filling stresses
e_{xx}, e_{yz} etc.	Filling strains
x, y, z	Geometrical coordinates (fig. 1)
u, v, w	Displacements in x, y, z directions
N_x, N_{xy}, N_y, Q_x, Q_y	Stress resultants of faces
M_x, M_{xy}, M_y	Stress couples of faces
$\bar{M}_x$, $\bar{M}_{xy}$, $\bar{M}_y$	Stress couples of whole plate
n, m	Number of buckling half wave lengths along and across the panel.

$\bar{n} = nb/a$ $\qquad \ell^2 = \frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}$ $\qquad \mu = 2h/t$

$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ $\qquad \varpi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ $\qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$\eta = \frac{L(1-\sigma^2)b^2}{\pi^2 E h t}$ $\qquad \xi = \frac{2C-L}{3C}\frac{\pi^2 h^2}{b^2}$ $\qquad \xi^x = \frac{L}{3C}\frac{\pi^2 h^2}{b^2}$

$D = \frac{Et^3}{12(1-\sigma^2)}$ $\qquad \rho = \frac{2b^2}{\pi^2 R(2h+t)}$ $\qquad \tau = \frac{E}{(1-\sigma^2)L}\cdot\frac{t}{R}$

$\kappa = \frac{L(1-\sigma^2)}{Et}$ $\qquad \nu = \frac{C+L}{C-L}$ $\qquad \varepsilon = 2C/L$

/ 3. SUMMARY ....

3. SUMMARY OF FORMULAE

(a) Buckling in Compression of Flat Panel

(i) Approximate Formulae for all Forms of Filling

1. Simply Supported Long Panel.

$$f_c = \frac{2(\mu + 1)^2 L}{\mu} \cdot \frac{\eta}{(1 + \eta)^2} = \frac{\pi^2 (2h + t)^2 E}{(1 - \sigma^2) b^2} \cdot \left(\frac{\eta}{1 + \eta}\right)^2 \qquad (26)^*$$

where $\eta/(\eta - 1) < b^2/200h^2$, $\eta = L(1 - \sigma^2)b^2/\pi^2 Eht$

$\mu = 2h/t$

2. Simply Supported Short Panel

Where $a/b = O(1)$ a correction factor to 1 is given at Fig. 10.

Where the panel is very wide compared to its length a modified Euler strut formula is obtained

$$f_c = \frac{\pi^2 (h + \frac{1}{2}t)^2 E}{a^2} \Big/ \left\{1 + \frac{\pi^2 Eht}{(1 - \sigma^2) L a^2}\right\} \qquad (27)$$

provided $a > 10h$

3. Long Panel with Clamped Edges

$$f_c = \frac{(\mu + 1)^2 L}{\mu} \cdot \frac{3.49\eta^2}{\eta^3 + 3.68\eta^2 + 3.6\eta - 1.3} \qquad (43)$$

$$= \frac{17.05(2h + t)^2 E}{(1 - \sigma^2) b^2} \cdot \frac{\eta^3}{\eta^3 + 3.68\eta^2 + 3.6\eta - 1.3}$$

1 and 3 are plotted at Fig.8 as correction factors to the equivalent plate stresses.
The buckling wave lengths are shown at Fig.9.

(ii) More Accurate Formula for Isotropic Filling (Simple Support)

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2} \left\{\frac{1}{\bar{n}^2 + \eta + 1 - \xi(\bar{n}^2 + 1)^2} + \frac{1}{3\eta(\mu + 1)^2}\right\} \qquad (24)$$

where $\bar{n}$ is chosen to make f_c a minimum and

$$\xi = \frac{2C - L}{3C} \cdot \frac{\pi^2 h^2}{b^2}$$

f_c/L versus μ is plotted for different η at Figs. 2 & 3 for $6\xi\eta/\mu = (1 - \sigma^2)(2C - L)L/EC$ equal to 0.001 and 0.0004 respectively.

/ (iii) Accurate .....

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* Numbers refer to equation numbers in main text.

(iii) Accurate Formula for Honeycomb Filling (Simple Support)

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2} \left\{ \frac{1}{\bar{n}^2 + \eta + 1 + \xi^x (\bar{n}^2 + 1)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\} \quad (34)$$

where $\bar{n}$ is chosen to make f_c a minimum and

$$\xi^x = \frac{L}{3C} \quad \frac{\pi^2 h^2}{b^2}$$

f_c/L versus μ is plotted for different η at Figs. 4 to 7 for values of $6\xi^x\eta/\mu = (1 - \sigma^2)L^2/EC$ of 2.5×10^{-4}; 10^{-4}; 2.5×10^{-5} and 10^{-5} respectively

) Wrinkling of a Simply Supported Flat Panel in Compression

(i) Isotropic Filling

$$\frac{f_c}{L} = \left\{ \frac{EC^2}{12(1 - \sigma^2)L(C + L)^2} \right\}^{\frac{1}{3}} \left\{ 1 + 2 \tanh\left[\mu \left(\frac{3(1 - \sigma^2)CL}{2E(C + L)} \right)^{\frac{1}{3}} \right] \right\} + \frac{2L}{C + L} \quad (30)$$

Approximate condition of validity is $\frac{\mu^3 CL}{E(C + L)} > 2$

This result is plotted at Figs. 2 and 3.

(ii) Aeolotropic Filling

$$\frac{f_c}{L} = \left\{ \frac{9EC(\sqrt{AC} - F)(\sqrt{AC} + 2L - F)}{16L^2(1 - \sigma^2)(\sqrt{AC} + L)^2} \right\}^{\frac{1}{3}} + \frac{\sqrt{AC} - F}{\sqrt{AC} + L} \quad (33)$$

Approximate condition of validity is

$$\frac{\mu^3(\sqrt{AC} - F)^2(\sqrt{AC} + 2L - F)^2}{ECL(\sqrt{AC} + L)} > 100$$

(iii) Honeycomb Filling

$$\frac{f_c}{L} = \sqrt{\frac{2EC}{3\mu(1 - \sigma^2)L^2}} = \frac{1}{\sqrt[3]{\xi^x \eta}} \quad (35)$$

This result is plotted at Figs. 4 to 6; the wrinkling stress is too high to appear on Fig. 7.

/ (c) Simply ....

(c) Simply Supported Curved Plates in Compression

(i) Panels

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu\bar{n}^2}\left\{\frac{(\bar{n}^2 + 1)^2}{\bar{n}^2 + \eta + 1} + \frac{\rho^2}{\eta}\right\} \quad (4$$

where $\bar{n}^2$ has the nearest value to $\dfrac{\eta + \overline{\eta + 1}\,\rho^2 + \eta\sqrt{\eta(\eta + \overline{\eta + 1}\,\rho^2)}}{\eta^2 - \eta - \rho^2}$,

$\rho = 2b^2/\pi^2 R(2h + t)$ and $\eta > \frac{1}{2}\left\{1 + \sqrt{1 + 4\rho^2}\right\}$

The factor of increase of this stress over that for a similar flat panel is plotted at Fig. 11.

(ii) Cylinders

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu}\left\{\frac{\bar{n}^2}{\bar{n}^2 + \eta} + \frac{\rho^2}{\eta\,\bar{n}^2}\right\} \quad (4$$

where $\bar{n}^2$ has the nearest value to $\rho\eta/(\eta - \rho)$ and $b = \pi R$

For long cylinders this reduces to:-

$$\frac{f_c}{L} = \tau\left\{1 + \mu - \tfrac{1}{2}\mu\tau\right\}, \text{ where } \tau = \frac{E}{(1 - \sigma^2)L}\cdot\frac{t}{R},$$

which is plotted at Fig. 12.

(d) Simply Supported Flat Panels in Shear

(i) Square Panels

$$\frac{q_c}{L} = \frac{4.72(\mu + 1)^2}{\mu\sqrt{(\eta + 8)(\eta + 5.19)}}, \ \eta > 10 \text{ say} \quad (5$$

or $$\frac{q_c}{E} = \frac{23.3(2h + t)^2}{(1 - \sigma^2)b^2}\cdot\frac{\eta}{\sqrt{(\eta + 8)(\eta + 5)}}$$

(ii) Long Panels

$$\frac{q_c}{L} = \frac{2.69(\mu + 1)^2}{\mu\eta}\left\{1 - \frac{3}{\eta} + \frac{5.8}{\eta^2}\right\} \quad (6$$

or $$\frac{q_c}{E} = \frac{13.3(2h + t)^2}{(1 - \sigma^2)b^2}\left\{1 - \frac{3}{\eta} + \frac{5.8}{\eta^2}\right\}$$

The shear stresses are plotted at Fig.13 as correction factor to the equivalent plate stresses. It is suggested that valu for intermediate values of the ratio b:a will be given by parabolic interpolation.

/4. General .....

4. GENERAL CONDITIONS OF EQUILIBRIUM

In this section the equations of equilibrium are derived that are required for finding the critical stresses of simply supported flat and curved panels under uniform compression by the method of considering neutral equilibrium under infinitesimal displacements.

The coordinates to be used are indicated in Fig. 1. They are cylindrical coordinates but instead of the usual r, θ, x for the radial, azimuthal and axial coordinates, $R + z$, y/R, x are used, where R is the radius of curvature. The sides and the direction of the applied load are parallel to the axis of the cylinder; the ends of the panel are cut by planes perpendicular to the axis.

The approach to the curved panel problem is to expand the various expressions involved in powers of 1/R and to neglect the higher powers. To this end we assume that R is large compared to the total panel thickness.

The faces are taken to be isotropic and the filling is assumed to be aeolotropic, but with symmetry about the normal to the plate. Writing X_x, Y_z etc. for the stresses and e_{xx}, e_{yz} etc. for the strains in the filling, the stress-strain relationship is given by the following scheme:-

$$\begin{aligned} X_x &= Ae_{xx} + (A - 2N)e_{yy} + Fe_{zz}, & Y_z &= Le_{yz} \\ Y_y &= (A - 2N)e_{xx} + Ae_{yy} + Fe_{zz}, & Z_x &= Le_{zx} \\ Z_z &= Fe_{xx} + Fe_{yy} + Ce_{zz}, & X_y &= Ne_{xy} \end{aligned} \qquad (1)$$

by writing u, v, w for the displacements in the x, y, z directions and neglecting second order terms in 1/R the well known equations relating strain and displacement in cylindrical coordinates become:-

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} \\ e_{yy} &= \frac{\partial v}{\partial y} - \frac{1}{R}\left\{z\frac{\partial v}{\partial y} - w\right\} \\ e_{zz} &= \frac{\partial w}{\partial z} \\ e_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{1}{R}\left\{z\frac{\partial w}{\partial y} + v\right\} \\ e_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ e_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{z}{R}\frac{\partial u}{\partial y} \end{aligned} \qquad (2)$$

/(b) Equilibrium ....

(b) Equilibrium of the Filling

In the same fashion the following equations of equilibrium are obtained:-

$$\frac{\partial X_x}{\partial x} + \frac{\partial Y_x}{\partial y} + \frac{\partial Z_x}{\partial z} - \frac{1}{R}\left\{z\frac{\partial Y_x}{\partial y} - Z_x\right\} = 0 \tag{3}$$

$$\frac{\partial X_y}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Z_y}{\partial z} - \frac{1}{R}\left\{z\frac{\partial Y_y}{\partial y} - 2Z_y\right\} = 0 \tag{4}$$

$$\frac{\partial X_z}{\partial x} + \frac{\partial Y_z}{\partial y} + \frac{\partial Z_z}{\partial z} - \frac{1}{R}\left\{z\frac{\partial Y_z}{\partial y} - Z_z + Y_y\right\} = 0 \tag{5}$$

On substituting from equations (1) and (2) into equations (3) and (4), we have:-

$$A\frac{\partial^2 u}{\partial x^2} + N\frac{\partial^2 u}{\partial y^2} + L\frac{\partial^2 u}{\partial z^2} + (A - N)\frac{\partial^2 v}{\partial x\partial y} + (F + L)\frac{\partial^2 w}{\partial x\partial z}$$
$$= \frac{1}{R}\left\{z\frac{\partial}{\partial y}\left[(A - N)\frac{\partial v}{\partial x} + 2N\frac{\partial u}{\partial y}\right] - (A - 2N - L)\frac{\partial w}{\partial x} - L\frac{\partial u}{\partial z}\right\} \tag{6}$$

and $$(A - N)\frac{\partial^2 u}{\partial x\partial y} + N\frac{\partial^2 v}{\partial x^2} + A\frac{\partial^2 v}{\partial y^2} + L\frac{\partial^2 v}{\partial z^2} + (F + L)\frac{\partial^2 w}{\partial y\partial z}$$
$$= \frac{1}{R}\left\{z\frac{\partial}{\partial y}\left[(A - N)\frac{\partial u}{\partial x} + 2A\frac{\partial v}{\partial y} + (F + L)\frac{\partial w}{\partial z}\right] - (A + 2L)\frac{\partial w}{\partial y} - L\frac{\partial v}{\partial z}\right\} \tag{7}$$

Write $\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, $\varpi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Differentiate equations (6) and (7) with respect to x and y respectively and add, so that:-

$$\left(A\nabla^2 + L\frac{\partial^2}{\partial z^2}\right)\delta + (F + L)\nabla^2\frac{\partial w}{\partial z}$$
$$= \frac{1}{R}\left\{z\frac{\partial}{\partial y}\left[2A\frac{\partial\delta}{\partial y} - (A - N)\frac{\partial\varpi}{\partial x} + (F + L)\frac{\partial^2 w}{\partial y\partial z}\right] - L\frac{\partial\delta}{\partial z}\right.$$
$$\left. - (A - 2N + L)\frac{\partial^2 w}{\partial x^2} - (A + 2L)\frac{\partial^2 w}{\partial y^2}\right\} \tag{8}$$

Differentiate equations (6) and (7) with respect to y and x respectively and subtract, so that:-

$$\left(N\nabla^2 + L\frac{\partial^2}{\partial z^2}\right)\varpi = \frac{1}{R}\left\{z\frac{\partial}{\partial y}\left[2N\frac{\partial\varpi}{\partial y} - (A - N)\frac{\partial\delta}{\partial y} - (F + L)\frac{\partial^2 w}{\partial y\partial z}\right]\right.$$
$$\left. - L\frac{\partial\varpi}{\partial z} + (2N + L)\frac{\partial^2 w}{\partial x\partial y}\right\} \tag{9}$$

/ Substituting .....

Substituting from equations (1) and (2) into equation (5) gives:-

$$(F + L)\frac{\partial\delta}{\partial z} + \left(L\nabla^2 + C\frac{\partial^2}{\partial z^2}\right) w$$

$$= \frac{1}{R}\left\{ z\frac{\partial}{\partial y}\left[(F + L)\frac{\partial v}{\partial z} + 2L\frac{\partial w}{\partial y}\right] + (L + A - F)\frac{\partial v}{\partial y} + (A - F - 2N)\frac{\partial u}{\partial x} - C\frac{\partial w}{\partial z} \right\} \quad (10)$$

Among the terms in $1/R^2$ neglected in the last equation is a term $\frac{A - F}{R^2}$ w which is worthy of note. It arises since e_{yy} contains w/R and is contained in $(Z_z - Y_y)/R$ in equation (5). When the buckling wavelength is of the same order as R, which it might be for a semicircular panel, $\nabla^2 w$ is of the same order as w/R^2. This would appear to limit the angle of the sector forming the panel for an accurate solution, but however these equations lead to solutions in section 8 which are identical to those obtained by other means.

(c) Equilibrium of the Faces

When considering the equilibrium of the faces it is necessary to make allowance for the fact that the stress resultants in the face and the reaction of the filling are not in the same plane.

Let unprimed quantities refer to values at an interface, $z = \pm h$, and primed quantities to values at the central surface of a face $z = \pm (h + \frac{1}{2}t)$ where 2h and t are the thickness of the filling and a face.

We have approximately:-

$$\left.\begin{aligned} u' &= u \mp \tfrac{1}{2}t\,\frac{\partial w}{\partial x} \\ v' &= v \mp \tfrac{1}{2}t\,\frac{\partial w}{\partial y} \\ w' &= w \pm \tfrac{1}{2}t\,\frac{\partial w'}{\partial z} \end{aligned}\right\}$$

where the top sign refers to the face $z = + h$ and the bottom to the face $z = - h$.

Hence $$\left.\begin{aligned} \delta' &= \delta \mp \tfrac{1}{2}t\nabla^2 w \\ \varpi' &= \varpi \end{aligned}\right\}$$

and since the normal stress in a face may be considered to be negligible $\sigma\frac{\partial u'}{\partial x} + \sigma\frac{\partial v'}{\partial y} + (1 - \sigma)\frac{\partial w'}{\partial z} = 0$

whence $w' = w \mp \frac{\sigma t}{2(1 - \sigma)}\,\delta'$

/ However ....

However since the areal dilatation is small compared to the transverse displacement the last correction can be ignored.

Equilibrium in the Surface of a Face

Neglecting terms in t/R and $1/R^2$ we have:-

$$\left.\begin{aligned} N_x &= \frac{Et}{1-\sigma^2}\left\{\frac{\partial u'}{\partial x} + \sigma\frac{\partial v'}{\partial y} - \frac{\sigma}{R}\left(\pm h\,\frac{\partial v'}{\partial y} - w'\right)\right\} \\ N_{xy} &= \frac{Et}{2(1+\sigma)}\left\{\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \mp \frac{h}{R}\,\frac{\partial u'}{\partial y}\right\} \\ N_y &= \frac{Et}{1-\sigma^2}\left\{\sigma\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} - \frac{1}{R}\left(\pm h\,\frac{\partial v'}{\partial y} - w'\right)\right\} \end{aligned}\right\} \qquad (11)$$

where N_x, N_{xy}, N_y are the stress resultants in the face and the top and bottom signs refer to face $z = +h$ and $-h$ respectively.

In considering the equilibrium of the stress resultants in the x and y direction account must be taken of the shear forces acting at the interfaces:-

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{h}{R}\,\frac{\partial N_{xy}}{\partial y} = \pm Z_x = \pm L\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)_{z=\pm h}$$

and

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} - \frac{h}{R}\,\frac{\partial N_y}{\partial y} = \pm Z_y + \frac{Q_y}{R} - \frac{Q_y}{R} \pm L\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \mp \frac{L}{R}\left(h\frac{\partial w}{\partial y} \mp v\right)_{z=\pm h} \qquad (12)$$

where Q_y is the y - stress resultant nrrmal to the face.

Substituting from equations (11) into equations (12) yields:-

$$\frac{\partial^2 u'}{\partial x^2} + \frac{1-\sigma}{2}\,\frac{\partial^2 u'}{\partial y^2} + \frac{1+\sigma}{2}\,\frac{\partial^2 v'}{\partial x\,\partial y} = \pm k\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$
$$\pm \frac{1}{R}\left\{\frac{1+\sigma}{2}\,h\,\frac{\partial^2 v'}{\partial x\partial y} \mp \sigma\frac{\partial w'}{\partial x} + (1-\sigma)h\,\frac{\partial^2 u'}{\partial y^2}\right\} \qquad (13)$$

and

$$\frac{1+\sigma}{2}\,\frac{\partial^2 u'}{\partial x\partial y} + \frac{1-\sigma}{2}\,\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2} = \pm k\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) - \frac{k}{R}\left(h\frac{\partial w}{\partial y} \pm v\right)$$
$$\pm \frac{1}{R}\left\{\frac{1+\sigma}{2}\,h\,\frac{\partial^2 u'}{\partial x\partial y} \mp \frac{\partial w'}{\partial y} + 2h\frac{\partial^2 v'}{\partial y^2} \pm \frac{k}{L}\,Q_y\right\} \qquad (14)$$

where $k = (1 - \sigma^2)L/Et$

/Differentiating ....

Differentiating equations (13) and (14) with respect to x and y respectively and adding, and eliminating the primed quantities, we obtain:-

$$\left(K\nabla^2 + \tfrac{1}{2}t\nabla^4\right)_w - \left(\pm\nabla^2 - K\frac{\partial}{\partial z}\right)_\delta = \frac{K}{R}\left(\pm h\frac{\partial^2 w}{\partial y^2} + \frac{\partial v}{\partial y}\right)$$
$$- \frac{1}{R}\left\{2h\frac{\partial^2 \delta}{\partial y^2} - \frac{1+\sigma}{2}h\frac{\partial^2 \varpi}{\partial x\partial y} \mp \sigma\frac{\partial^2 w}{\partial x^2} \mp \frac{\partial^2 w}{\partial y^2} \pm \frac{K}{L}\frac{\partial Q_y}{\partial y}\right\} \quad (15)$$

Differentiating equations (13 and (14) with respect to y and x and subtracting, we obtain:-

$$\frac{1-\sigma}{2}\nabla^2\varpi \mp K\frac{\partial\varpi}{\partial z} = \pm\frac{1}{R}\left\{(1-\sigma)h\frac{\partial^2\varpi}{\partial y^2} - \frac{1+\sigma}{2}h\frac{\partial^2\delta}{\partial x\partial y}\right.$$
$$\left.\pm(1-\sigma)\frac{\partial^2 w}{\partial x\partial y} \mp \frac{K}{L}\frac{\partial Q_y}{\partial x}\right\} \quad (16)$$

Equilibrium Normal to a Face

With the same approximations the stress couples, M_x, M_{xy}, M_y, are given by:-

$$\left.\begin{aligned}
M_x &= -D\left\{\frac{\partial^2 w}{\partial x^2} + \sigma\frac{\partial^2 w}{\partial y^2} \mp \frac{\sigma}{R}\left(2h\frac{\partial^2 w}{\partial y^2} \pm \frac{\partial v}{\partial y}\right)\right\}\\
M_{xy} &= -(1-\sigma)D\left\{\frac{\partial^2 w}{\partial x\partial y} \mp \frac{1}{R}\left(h\frac{\partial^2 w}{\partial x\partial y} \pm \frac{\partial v}{\partial x}\right)\right\}\\
M_y &= -D\left\{\sigma\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \mp \frac{1}{R}\left(2h\frac{\partial^2 w}{\partial y^2} \pm \frac{\partial v}{\partial y}\right)\right\}
\end{aligned}\right\} \quad (17)$$

where $D = Et^3/12(1-\sigma^2)$

In considering the equilibrium of the shears and couples account must be taken of the distributed moment of the shear forces at the interfaces about the centre of the faces:-

$$\left.\begin{aligned}
Q_x &= \frac{\partial M_x}{\partial x} + \left(1 \mp \frac{h}{R}\right)\frac{\partial M_{xy}}{\partial y} + \tfrac{1}{2}t\,Z_x\\
Q_y &= \frac{\partial M_{xy}}{\partial x} + \left(1 \mp \frac{h}{R}\right)\frac{\partial M_y}{\partial y} + \tfrac{1}{2}t\,Z_y
\end{aligned}\right\} \quad (18)$$

Similarly in considering the equilibriums of the shears and normal forces account must be taken of the tension the filling exerts on the faces:-

$$\frac{\partial Q_x}{\partial x} + \left(1 \mp \frac{h}{R}\right)\frac{\partial Q_y}{\partial y} = ft\frac{\partial^2 w}{\partial x^2} + \frac{N_y}{R} \pm Z_z$$

where f is the stress in the faces resulting from the applied load.

/On substituting ....

On substituting from equations (17) and (18) into the last equation we obtain:-

$$
\begin{aligned}
& ft\frac{\partial^2 w}{\partial x^2} + D\nabla^4 w - \tfrac{1}{2}tL\left(\nabla^2 w + \frac{\partial\delta}{\partial z}\right) \pm C\frac{\partial w}{\partial z} \pm F\delta \\
& = \frac{D}{R}\left\{\pm 4h\,\nabla^2\frac{\partial^2 w}{\partial y^2} + (2-\sigma)\frac{\partial^3 v}{\partial x^2\partial y} + \frac{\partial^3 v}{\partial y^3}\right\} \\
& \mp \frac{\tfrac{1}{2}tL}{R}\left\{2h\frac{\partial^2 w}{\partial y^2} + h\frac{\partial^2 v}{\partial y\,\partial z} \pm \frac{\partial v}{\partial y}\right\} + \frac{F}{R}\left\{h\frac{\partial v}{\partial y} \mp w\right\} \\
& - \frac{Et}{(1-\sigma^2)R}\left\{\sigma\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{w}{R}\right\}
\end{aligned}
\tag{19}
$$

It will be noted that the last term in the last bracket of this equation is of order $1/R^2$. This term is retained because it is not a derivative and because E is so much greater than the other elastic constants; other possible terms that might be retained are either derivatives which are necessarily smaller for long wave buckling, or contain elastic constants of the filling.

/5. Simply ....

5. SIMPLY SUPPORTED FLAT PANEL IN UNIFORM COMPRESSION

The conditions of equilibrium are given by putting $1/R = 0$ in equations (8), (9), (10), (15), (16) and (19) and they are:-

$$\left.\begin{aligned} \left(A\nabla^2 + L\frac{\partial^2}{\partial z^2}\right)\delta + (F+L)\nabla^2\frac{\partial w}{\partial z} &= 0 \\ (F+L)\frac{\partial \delta}{\partial z} + \left(L\nabla^2 + C\frac{\partial^2}{\partial z^2}\right)w &= 0 \end{aligned}\right\} \qquad (20)$$

$$\left.\begin{aligned} \left(N\nabla^2 + L\frac{\partial^2}{\partial z^2}\right)\varpi &= 0 \\ \left(\frac{1-\sigma}{2}\nabla^2 \mp K\frac{\partial}{\partial z}\right)\varpi &= 0 \text{ at } z = \pm h \end{aligned}\right\} \qquad (21)$$

$$\left.\begin{aligned} (K\nabla^2 + \tfrac{1}{2}t\nabla^4)w - \left(\pm\nabla^2 - K\frac{\partial}{\partial z}\right)\delta &= 0 \\ ft\frac{\partial^2 w}{\partial x^2} + D\nabla^4 w - \tfrac{1}{2}tL\left(\nabla^2 w + \frac{\partial \delta}{\partial z}\right) \pm C\frac{\partial w}{\partial z} \pm F\delta &= 0 \end{aligned}\right\} \text{ at } z = \pm h \qquad (22)$$

If we assume w to be of the form $\Sigma W_{mn}(z)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$ (n, m integers), δ will be a similar series while ϖ will be a double cosine series.

Since the equations containing ϖ do not contain w or δ, the coefficients in the series for ϖ will be independent of those for w and δ. In each coefficient of ϖ there will be two constants of integration which must be chosen to satisfy the second equation of (21) at z = +h and -h. It is clear that the only possible solution is $\varpi = 0$.

Each pair of corresponding terms in the two series for w and δ must satisfy equations (20) and (22) separately. In solving equations (20) each pair of coefficients will contain four constants of integration, which must be chosen to satisfy the four equations of (22). The latter requirement will determine the value of f, but f will now be a function of m and n, so that the equations cannot be satisfied for all m, n at the same time. Hence w will consist of one term only, instead of a series, corresponding to one pair, m and n, which must be chosen to make f least.

/For a panel ......

For a panel with edges at $x = 0, a$ and $y = 0, b$ we will have $w = 0$ at the edges. The bending moments of the plate as a whole will also be zero at the edges since in addition to $\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0$ we have $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$, because $\delta = 0$ at the edges and $\sigma = 0$ everywhere, making the direct stresses as well as the bending moments in the faces vanish at the boundaries. Hence the conditions of simple support are satisfied apart from small displacements in the plane of the panel.

<u>Isotropic Filling</u>

For an isotropic filling $A = C$, $N = L$, $F = C - 2L$

The solution of equations (20) is:-

$$w = (W_1 c + W_2 \ell z s + W_1' s + W_2' \ell z c) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

$$\delta = -\left\{W_1 s + W_2(\ell z c + \nu s) + W_1' c + W_2'(\ell z s + \nu c)\right\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

where $c = \mathrm{ch}\,\ell z$, $s = \mathrm{sh}\,\ell z$

$$\ell^2 = \frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}$$

$$\nu = (C + L)/(C - L)$$

W_1, W_2, W_1', W_2' = consts. of integration

Now w and δ are the sums of even and odd functions of z: to the even part of w corresponds the odd part of δ and vice versa. It will be seen from inspection of equations (22), taking due account of the alternative signs, that the odd and even parts of w will form independent solutions.

Let us first consider the even part of w (i.e. $W_1' = W_2' = 0$) corresponding to the buckling of the panel as a whole or "in-phase buckling".

On substituting for w and δ in equations (22), we have:-

$$\frac{W_1}{W_2} = -\frac{\{\kappa(1 + \nu) + \ell^2 h\}c + \{2\kappa h + \nu - \frac{1}{2}th\ell^2\}\ell s}{(2\kappa - \frac{1}{2}t\ell^2)c + \ell s}$$

$$\& \quad \frac{n^2\pi^2}{a^2} ft = D\ell^4 + \frac{L\{t\ell c + 2s\}\ell W_1 + \{L(2h + \frac{1+\nu}{2}t)\ell c + (C - \nu F + th\ell^2 L)s\}\ell W_2}{W_1 c + W_2 \ell h s}$$

$$= D\ell^4 + \frac{\ell L\left\{\begin{array}{l}\left[\{2(C - L)th + \frac{1}{2}Ct^2\}\ell^2 - 4\kappa h(C - L)\right]\ell c^2 \\ + 2\left[2\kappa(C - L) + t\ell^2 L\right] cs \\ + 2\left[2\kappa h(C - L) + C - (C - L)th\ell^2\right]\ell s^2\end{array}\right\}}{\left[2\kappa C + (C - L)\ell^2 h\right]c^2 + (C + L)\ell cs - (C - L)h\ell^2 s^2} \qquad (23)$$

/In order ....

In order to minimise the expression for f it is necessary to make an approximation. Consider first long wave buckling in which ℓh is small, say < 0.7, so that approximately $\tanh \ell h = \ell h - \frac{1}{3}\ell^3 h^3$ Neglecting higher powers of ℓh the equation reduces to:-

$$\frac{n^2\pi^2}{a^2}ft = \ell^4\left\{\frac{(h + \frac{1}{2}t)^2 + \frac{1}{3}h^3(4K\frac{C - L}{C} - \frac{L}{C}t\ell^2)}{K + \ell^2 h - \frac{1}{3}\ell^4 h^3(2 - \frac{L}{C})} L + D\right\}$$

In general the second bracket of the numerator of the fraction on the right hand side is small, so the equation reduces approximately to:-

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu}\cdot\frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2}\left\{\frac{1}{\eta + \bar{n}^2 + m^2 - \xi(\bar{n}^2 + m^2)} + \frac{1}{3\eta(\mu + 1)^2}\right\} \qquad (24)$$

where $\mu = 2h/t$

$$\eta = \frac{Kb^2}{\pi^2 h} = \frac{(1 - \sigma^2)L\, b^2}{\pi^2\, Eht}$$

$$\xi = \frac{2C - L}{3C}\cdot\frac{\pi^2 h^2}{b^2}$$

$$\bar{n} = nb/a$$

Now $\frac{\partial f}{\partial m} - \frac{\partial f}{\partial n} = \frac{2}{\bar{n}} f > 0$, so that, if there exist m and n for which $\frac{\partial f}{\partial n}$ vanishes, the least f occurs at the least possible value of m, which in this case is unity.

The minimum value f_c/L of this expression is plotted against μ for a range of η of 0.2 to 20 for $6\xi\eta/\mu = (1-\sigma^2)(2C-L)L/EC$ equal to 0.001 at Fig.2 and equal to 0.0004 at Fig.3 In computing these curves it is assumed that the panel is long so that $\bar{n} = nb/a$ can take up its optimum value.

For $\eta > 1$ a further approximation may be made:-

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu}\cdot\frac{(\bar{n}^2 + 1)^2}{\bar{n}^2(\bar{n}^2 + 1 + \eta)} \qquad (25)$$

This gives for a long panel, $\bar{n} = \sqrt{\frac{\eta + 1}{\eta - 1}}$ and:-

$$\frac{f_c}{L} = \frac{2(\mu + 1)^2}{\mu}\cdot\frac{\eta}{(\eta + 1)^2} \qquad (26)$$

This expression requires a correction when the panel is not long to allow for $\bar{n}$ being restricted by virtue of n being an integer. The correction is plotted at Fig.10.

/ The last ....

The last approximation is tantamount to substituting ℓh for $\tanh \ell h$ so that a reasonable condition for validity is $\frac{\eta}{\eta - 1} < \frac{b^2}{200h^2}$

When b is large compared to a, n = 1 so equation 25 becomes:-

$$f_c = \frac{(\mu + 1)^2 L}{2\mu} \frac{\left[\left(\frac{b}{a}\right)^2 + 1\right]^2}{\left(\frac{b}{a}\right)^2 \cdot \left[(1 + \eta) + \left(\frac{b}{a}\right)^2\right]}$$

$$\doteq \frac{(\mu + 1)^2 L b^2}{2\mu a^2 \eta} \Big/ \left(1 + b^2/a^2\eta\right)$$

$$\text{so } f_c = \frac{\pi^2(h + \frac{1}{2}t)^2 E}{a^2} \Big/ \left(1 + \frac{\pi^2 Eht}{(1 - \sigma^2)La^2}\right) \tag{27}$$

which is a modified Euler strut formula.

Now consider the opposite approximation in which ℓh is large, say > 2, so that $\tanh \ell h \doteq 1$ corresponding to short wave buckling, or wrinkling of the faces. This rather drastic approximation implies that the two faces fail independently.
With $\tanh \ell h = 1$ equation (23) yields:-

$$\frac{n^2\pi^2}{a^2} ft = D\ell^4 + \frac{L\ell\{4K(C - L) + 2C\ell + 2tL\ell^2 + \frac{1}{2}Ct^2\ell^3\}}{2KC + (C + L)\ell}$$

As before m = 1; also $\ell^2 = \frac{\pi^2}{b^2} + \frac{n^2\pi^2}{a^2} > \frac{4}{h^2}$ by our assumption, but $\frac{4}{h^2}$ is in general large compared to $\frac{\pi^2}{b^2}$ so that $\frac{n\pi}{a}$ can be replaced by ℓ.

In addition $K/\ell < h\ell/2tE < 0.01$ say, so that terms in K can be neglected. The ratio of the fourth term of the numerator of the faction to the third, which in itself is small, is $\frac{1}{4}\sqrt[3]{\frac{L}{E}}$ approximately and may also be neglected.

Therefore:-

$$ft = D\ell^2 + \frac{2CL}{(C + L)\ell} + \frac{2tL^2}{C + L} \tag{28}$$

Hence:-

$$\frac{f_c}{L} = \sqrt[3]{\frac{9EC^2}{4(1 - \sigma^2)L(C + L)^2}} + \frac{2L}{C + L}$$

for which $\frac{n}{a} = \frac{2}{\pi t}\sqrt[3]{\frac{3(1 - \sigma^2)\,CL}{2E(C + L)}}$.

/ This makes .....

This makes $\ell h \simeq \frac{2L}{t} \sqrt[3]{\frac{L}{E}}$ which may well be as low as 1, in which case the approximation made would not be acceptable. From an inspection of equation (23) it would appear that a better approximation in place of equation (28) might be:-

$$ft = D\ell^2 + \frac{2CL}{C+L} \cdot \frac{\tanh \ell h}{\ell} + \frac{2tL^2}{C+L} \tag{29}$$

Using the previous value of ℓ we have:-

$$\frac{f_c}{L} = \left\{\frac{EC^2}{12(1-\sigma^2)L(C+L)^2}\right\}^{\frac{1}{3}} \left\{1 + 2\tanh\left[\left(\frac{2h}{t}\,\frac{3(1-\sigma^2)CL}{2E(C+L)}\right)^{\frac{1}{3}}\right]\right\} + 2L/(C+L) \tag{30}$$

This second approximation will be inaccurate if $\ell h < 3/2$, so $\mu^3 CL/E(C+L) > 2$ is an approximate condition of validity.

It should be noted that $\eta = 1$ is not the threshold between long wave buckling and wrinkling, but only the boundary of validity of expression (26). Though indeed the wavelength shortens as η tends to zero, the expression $2b\sqrt{\frac{\eta - 1}{\eta + 1}}$ for the wavelength no longer holds as η approaches one.

We will now show that out of phase buckling modes do not occur; that is modes with w as an odd function of z. By repeating the same process as for in phase buckling an equation like (23) will be derived except that $ch\ell h$ and $sh\ell h$ will be interchanged.

Considering long wave buckling first, put $\tanh \ell h = \ell h - \frac{1}{3}\ell^2 h^2$ as before, giving:-

$$\frac{n^2\pi^2}{a^2}ft = D\ell^4 + L\,\frac{C + 4\kappa h(C-L) - h\ell^2\left[(C-L)t + \frac{8}{3}\kappa h^2(C-L)\right]}{h\left[L + \kappa hC + \frac{1}{3}\ell^2 h^2(C - 2L - \kappa hC)\right]} \tag{31}$$

As a first approximation write:-

$$\frac{n^2\pi^2}{a^2}\,ft = D\ell^4 + C/h$$

/ so that ....

so that for a minimum $\left(\frac{\pi n}{a}\right)^4 = \left(\frac{\pi}{b}\right)^4 + C/Dh$ whence $(h\ell)^4 >$ $\left(\frac{\pi nh}{a}\right)^4 > Ch^3/D = 12(1-\sigma^2)\left(\frac{h}{t}\right)^3 \frac{C}{E}$ which in general gives a greater value of ℓh than the approximation will stand. Now the full expression (31) will yield an even greater value of ℓh, since its second term on the right hand side is a decreasing function of ℓ instead of a constant as first assumed. It is therefore concluded that longwave out of phase buckling does not arise.

With regard to short wave out of phase buckling we will obtain the same expression as for in phase buckling on putting $\tanh \ell h = 1$ for the first approximation. However for the second approximation the result of interchanging $ch\ell h$ and $sh\ell h$ will be the counterpart to equation (29) with $\coth \ell h$ for $\tanh \ell h$, clearly leading to a greater critical stress.

Aeolotropic Filling

The solution to the equilibrium equations (20) for the filling are:-

$$\left.\begin{aligned} w &= \left\{W_1c_1 + W_2c_2 + W_1's_1 + W_2's_2\right\}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} \\ \delta &= \ell\left\{\Delta_1 s_1 + \Delta_2 s_2 + \Delta_1' s_1 + \Delta_2' s_2\right\}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} \end{aligned}\right\}$$

where $c_1 = ch\, k_1 \ell z$, $s_1 = sh\, k_1 \ell z$ etc.

$$\ell^2 = \frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}$$

k_1, k_2 are the roots of $LCk^4 - (AC - 2LF - F^2)k^2 + AL = 0$

$W_1/\Delta_1 = W_1'/\Delta_1' = (F + L)k_1/(L - k_1^2C)$ etc.

Proceeding as for isotropic filling and considering only the even part of w, we have on substituting for w and δ into equations (22):-

$$W_1c_1(K - \tfrac{1}{2}t\ell^2) - \Delta_1(\ell s_1 + Kk_1c_1)$$

$$= - W_2c_2(K - \tfrac{1}{2}t\ell^2) + \Delta_2(\ell s_2 + Kk_2c_2)$$

$$\text{and } \frac{n^2\pi^2}{a^2}ft = D\ell^4 + \left\{\frac{\ell\left[(Ck_1W_1 + F\Delta_1)s_1 + (Ck_2W_2 + F\Delta_2)s_2\right] + \tfrac{1}{2}tL\ell^2\left[(W_1 - k_1\Delta_1)c_1 + (W_2 - k_2\Delta_2)c_2\right]}{W_1c_1 + W_2c_2}\right\}$$

/ whence .....

whence:-

$$\frac{W_1}{W_2} = -\frac{k_1}{k_2}\frac{\left(\kappa F - \frac{1}{2}t\ell^2(F + L) + \kappa k_2{}^2C\right)k_2c_2 + \left(k_2{}^2\ C - L\right)\ell s_2}{\left(\kappa F - \frac{1}{2}t\ell^2(F + L) + \kappa k_1{}^2C\right)k_1c_1 + \left(k_1{}^2\ C - L\right)\ell s_1}$$

$$\text{and } \frac{n^2\pi^2}{a^2}\,ft = D\ell^4 + \frac{\ell L}{F + L}\frac{\left\{\begin{array}{l}\left[C\left(k_1W_1s_1 + k_2W_2s_2\right) + F\left(W_1s_1/k_1 + W_2s_2/k_2\right)\right] \\ + \frac{1}{2}t\,\ell\left[C\left(k_1{}^2W_1c_1 + k_2{}^2W_2c_2\right) + F\left(W_1c_1 + W_2c_2\right)\right]\end{array}\right\}}{W_1\ c_1\ +\ W_2\ c_2} \tag{32}$$

As an approximation for longwave buckling (ℓh small) put $c_1 = c_2 = 1$, $s_1 = k_1\ell h$ and $s_2 = k_2\ell h$ so that:-

$$\frac{k_1{}^2W_1 + k_2{}^2W_2}{W_1 + W_2} = -\frac{\kappa F - \frac{1}{2}t\ell^2(F + L) - \ell^2 hL}{(\kappa + \ell^2h)C}$$

$$\text{and } \frac{n^2\pi^2}{a^2}ft = \frac{(h + \frac{1}{2}t)L\ell^2}{F + L}\left\{\frac{k_1{}^2W_1 + k_2{}^2W_2}{W_1 + W_2}\ C + F\right\}$$

$$= \frac{(h + \frac{1}{2}t)^2\,\ell^4\,L}{\kappa + \ell^2h}$$

i.e. $\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu}\cdot\frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2(\bar{n}^2 + m^2 + \eta)}$, which is the same expression as derived for an isotropic filling (equation 25) by a comparable approximation.

For short wave buckling (ℓh large) put $c_1 = c_2 = s_1 = s_2$ so that equations (32) become:-

$$\frac{W_1}{W_2} = -\frac{k_1}{k_2}\cdot\frac{\left(\kappa F - \frac{1}{2}t\ell^2(F + L) + \kappa k_2{}^2C\right)k_2 + \left(k_2{}^2C - L\right)\ell}{\left(\kappa F - \frac{1}{2}t\ell^2(F + L) + \kappa k_1{}^2C\right)k_1 + \left(k_1{}^2C - L\right)\ell}$$

$$\text{and } \frac{n^2\pi^2}{a^2}ft = D\ell^4 + \frac{\ell L}{F+L}\frac{\left\{\begin{array}{l}C\left(k_1W_1 + k_2W_2\right) + F\left(\frac{W_1}{k_1} + \frac{W_2}{k_2}\right) \\ + \frac{1}{2}t\,\ell\left(k_1{}^2W_1 + k_2{}^2W_2 + F\left(W_1 + W_2\right)\right)\end{array}\right\}}{W_1\ +\ W_2}$$

$$= D\ell^4 + \frac{\ell L}{F + L}\frac{\left\{\begin{array}{l}\kappa F^2 + FC\left(k_1{}^2 + k_2{}^2\right) + k_1{}^2k_2{}^2C + \ell C(F + L)\left(k_1 + k_2\right) \\ + \frac{1}{2}t\ell^2\left[LC\left(k_1{}^2 + k_2{}^2\right) - C^2k_1{}^2k_2{}^2 - 2C(F + L)k_1k_2 - F^2\right] \\ + \frac{1}{2}t\ell^3\ C\ (F + L)\ k_1k_2\ (k_1 + k_2)\end{array}\right\}}{\ell\left(Ck_1k_2 + L\right) + \kappa C\ k_1k_2\ \left(k_1 + k_2\right)}$$

/ For similar ....

For similar reasons to those given for short wave buckling with an isotropic filling, put $\frac{n\pi}{a} = \ell$ and neglect terms in κ and $t^2\ell^3$, so that:-

$$ft = D\ell^2 + \frac{LC(k_1+k_2)}{\ell(Ck_1k_2+L)} + \frac{\frac{1}{2}tL\left[LC(k_1{}^2+k_2{}^2)+2(F+L)Ck_1k_2-k_1{}^2k_2{}^2C^2-F^2\right]}{(F+L)(Ck_1k_2+L)}$$

which is least for $\frac{n\pi}{a} = \ell = \sqrt[3]{\frac{LC(k_1+k_2)}{2D(Ck_1k_2+L)}}$, so that

$$\frac{f_c}{L} = \sqrt[3]{\frac{9EC(\sqrt{AC}-F)(\sqrt{AC}+2L+F)}{16L^2(1-\sigma^2)(\sqrt{AC}+L)^2}} + \frac{\sqrt{AC}-F}{\sqrt{AC}+L} \qquad (33)$$

on substituting for k_1, k_2 from the quadratic in k^2.

This formula only holds when $k_1\ell h$ and $k_2\ell h$ are large, say greater than 2, so that an approximate condition of validity is:-

$$\frac{\mu^3(\sqrt{AC}-F)^2(\sqrt{AC}+2L+F)^2}{ELC(\sqrt{AC}+L)} > 100$$

since $(k_1\ell h)^3$ or $(k_2\ell h)^3 \simeq \left[\frac{(k_1+k_2)\ell h}{2}\right]^3 = \frac{h^3LC(k_1+k_2)^4}{16D(Ck_1k_2+L)}$

Honeycomb Filling

It is assumed that this type of filling has no stiffness in the plane of the plate, so that $A = N = F = 0$. The solution to the equations of equilibrium of the filling now become:-

$$w = \left\{W_1 + W_2\ell^2z^2 + W_2{}'\ell z\right\}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

$$\delta = \ell\left\{W_1\ell z - W_2\left(\varepsilon\ell z - \tfrac{1}{3}\ell^3z^3\right) + W_1{}' + \tfrac{1}{2}W_2{}'\ell^2z^2\right\}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

where $\varepsilon = 2C/L$

Proceeding as in the previous two cases for in-phase buckling we obtain on substituting for w and δ in equations (22)

$$\frac{W_1}{W_2} = \frac{\varepsilon(\kappa + H\ell^2) - (\frac{1}{3}h + \frac{1}{2}t)h^2\ell^4}{(h+\frac{1}{2}t)\ell^2}$$

and $$\frac{n^2\pi^2}{a^2}ft = D\ell^4 + \frac{LW_2\ell^2\varepsilon(h+\frac{1}{2}t)}{W_1 + W_2\ell^2h^2}$$

$$= \ell^4\left\{\frac{(h+\frac{1}{2}t)^2}{\kappa + \ell^2h + \frac{L}{3C}\ell^4h^3}L + D\right\}$$

/ whence ....

whence $\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2} \left\{ \frac{1}{\eta + \bar{n}^2 + m^2 + \xi^x(\bar{n}^2 + m^2)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\}$

$$\text{where } \xi^x = \frac{L}{3C} \frac{\pi^2 h^2}{b^2} \qquad (34)$$

This is of the same form as obtained for an isotropic filling (24) and approximates to precisely the same expression (25) obtained for the other two types of material for long wave buckling when $\eta > 1$.

When $\bar{n}^2$ is large compared to $(1 + \eta)$, an alternative approximation may be made:-

$$\frac{f}{L} \simeq \frac{(\mu + 1)^2}{2\mu} \left\{ \frac{1}{1 + \xi^x \bar{n}^2} + \frac{\bar{n}^2}{3\eta(\mu + 1)^2} \right\}$$

so that $\frac{f_c}{L} = \frac{1}{\mu} \left\{ \frac{\mu + 1}{\sqrt{3\eta\xi^x}} - \frac{1}{6\eta\xi^x} \right\}$

for which $\bar{n}^2 = \frac{1}{\xi^x} \left\{ \overline{\mu + 1} \sqrt{3\eta\xi^x} - 1 \right\}$

This approximate form corresponds to an in-phase wrinkling. It does not necessarily give the critical stress since the full expression may have more than one minimum.

The critical stress computed from the full expression is plotted as f_c/L versus μ for the range of η of 0.1 to 20 at Figs 4 to 7 for values of $6\xi^x\eta/\mu = (1 - \sigma^2)L^2/EC$ of 2.5×10^{-4}, 10^{-4}, 2.5×10^{-5} and 10^{-5} respectively.

For out of phase buckling we consider the odd part of w i.e. $W_2 \ell z \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$ and obtain directly:-

$$\frac{n^2\pi^2}{a^2} ft = D\ell^4 + C/h$$

so that $f_c t = \frac{2\pi^2}{b^2} D \left\{ 1 + \sqrt{1 + \frac{Cb^4}{\pi^4 Dh}} \right\} \simeq 2\sqrt{\frac{DC}{h}}$

for which $\left(\frac{n\pi}{a}\right)^4 = \frac{\pi^4}{b^4} + \frac{C}{Dh}$

$$\text{Hence } \frac{f_c}{L} = \sqrt{\frac{tEC}{3h(1 - \sigma^2)L^2}} = \frac{1}{3\sqrt{\xi^x \eta}} \qquad (35)$$

/ This mode ....

This mode is short wave buckling or wrinkling and is shown as the limiting lines on Figs 4 to 6.

These results for honeycomb filling are basically those obtained by Hemp (ref. 2), who made use of the fact that for such a material Z_x and Z_y are functions of x and y only (see equations 3 & 4), thereby being enabled to integrate the equations of equilibrium of the filling directly.

It will have been noted that the displacements of a filling of isotropic or aeolotropic material contain hyperbolic functions of z while those of a honeycomb filling do not. This indicates that a disturbance initiated at the boundary of a large mass of the former would be dissipated while that in a honeycomb material would be transmitted indefinitely. However unreal the latter may appear, the possibility of the elastic form considered being a fair approximation to the truth is not precluded when confined to plates of no great thickness.

6. SIMPLIFIED MECHANISM OF LONG WAVE BUCKLING

The most important result so far obtained is that over a great range of conditions the long wave buckling stress of flat panels in compression is independent of the form of the filling: of its elastic constants only the transverse shear modulus is of concern. It will now be shown that the approximation leading to the common formula for the critical stress is equivalent to the replacement of the actual filling by an idealised form, allowing the development of equations treating the plate as a whole. This concept greatly facilitates the handling of problems where the use of double Fourier series becomes unwieldy.

In the theory of thin plates it is usual to consider the central surface as inextensible and that fibres initially perpendicular to it remain so. This approach, which in particular neglects shear deformation, results in the equivalent plate theory when applied to sandwich plates. If we denote by f_e the failing stress according to the latter theory of a simply supported panel in compression, we have

$f_e = \frac{\pi^2 E(2h + t)^2}{(1 - \sigma^2)\, b^2}$ so that we can rewrite the formula

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{\mu} \cdot \frac{2\eta}{(1 + \eta)^2} \quad \text{as} \quad f_c = \left(\frac{\eta}{1 + \eta}\right)^2 f_e$$

If we denote by P_s the shear stiffness, 2hL, of the filling and by $P_e = 2f_e t$ the strength of the panel according to equivalent plate theory, then $\eta \simeq 4P_s/P_e$, so that parameter, η, is a measure of the shear stiffness in terms of the strength of the plate as a whole and also of the accuracy of the equivalent plate theory.

It is therefore natural to modify the equivalent plate theory by some such device as assuming that the fibres initially normal to the neutral surface remain straight but deflected by the action of shear forces, necessarily assumed uniform across the filling. This leads to the following formulae for the displacements of the central surfaces of the faces:-

$$\begin{aligned} u &= -(h + \tfrac{1}{2}t)\frac{\partial w}{\partial x} + \frac{hZ_x}{L} \\ v &= -(h + \tfrac{1}{2}t)\frac{\partial w}{\partial y} + \frac{hZ_y}{L} \end{aligned} \qquad (36)$$

where the transverse displacement, w, and the filling shear stress, Z_x and Z_y are assumed constant along any normal

/ Hence ....

Hence, neglecting the stiffness of the faces themselves, we have:-

$$\left.\begin{aligned}\bar{M}_x &= -\frac{(2h+t)Et}{1-\sigma^2}\left\{(h+\tfrac{1}{2}t)\left[\frac{\partial^2 w}{\partial x^2}+\sigma\frac{\partial^2 w}{\partial y^2}\right]-\frac{h}{L}\left[\frac{\partial Z_x}{\partial x}+\sigma\frac{\partial Z_y}{\partial y}\right]\right\}\\ \bar{M}_{xy} &= -\frac{(2h+t)Et}{2(1+\sigma)}\left\{2(h+\tfrac{1}{2}t)\frac{\partial^2 w}{\partial x\partial y}-\frac{h}{L}\left[\frac{\partial Z_x}{\partial y}+\frac{\partial Z_y}{\partial x}\right]\right\}\\ \bar{M}_y &= -\frac{(2h+t)Et}{1-\sigma^2}\left\{(h+\tfrac{1}{2}t)\left[\sigma\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]-\frac{h}{L}\left[\sigma\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}\right]\right\}\end{aligned}\right\} \quad (37)$$

where $\bar{M}_x$, $\bar{M}_{xy}$, $\bar{M}_y$, are the stress couples of the sandwich as a whole.

For equilibrium:-

$$\left.\begin{aligned}(2h+t)Z_x &= \frac{\partial \bar{M}_x}{\partial x}+\frac{\partial \bar{M}_{xy}}{\partial y}\\ \text{and}\quad (2h+t)Z_y &= \frac{\partial \bar{M}_{xy}}{\partial x}+\frac{\partial \bar{M}_y}{\partial y}\end{aligned}\right\}$$

$$\left.\begin{aligned}&\text{so that}\ \left(\nabla^2-\frac{\pi^2\eta}{b^2}\right)\left(\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}\right) = \frac{L(\mu+1)}{\mu}\nabla^4 w\\ &\text{and}\ \left(\frac{1-\sigma}{2}\nabla^2-\frac{\pi^2\eta}{b^2}\right)\left(\frac{\partial Z_x}{\partial y}-\frac{\partial Z_y}{\partial x}\right) = 0\end{aligned}\right\} \quad (38)$$

Now for a plate in compression the condition for equilibrium of the normal forces is:-

$$2ft\,\frac{\partial^2 w}{\partial x^2} = (2h+t)\left(\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}\right) \quad (39)$$

which on eliminating $\left(\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}\right)$ gives

$$f\left(\nabla^2-\frac{\pi^2\eta}{b^2}\right)\frac{\partial^2 w}{\partial x^2} = \frac{L(\mu+1)^2}{2\mu}\nabla^4 w \quad (40)$$

If $w = W\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$ then from equation (38) it follows that $\frac{\partial Z_x}{\partial x}$ and $\frac{\partial Z_y}{\partial y}$ are also proportioned to $\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$. Hence by equations (37) $\bar{M}_x = \bar{M}_y = 0$ as well as $w = o$ at the edges $x = o,a$ and $y = o,b$ of the panel, satisfying the boundary conditions for a simply supported panel.

Substituting for w in equation (40) gives

$$\frac{f}{L} = \frac{(\mu+1)^2}{2\mu}\cdot\frac{\left(\bar{n}^2+m^2\right)^2}{\bar{n}^2\left(\bar{n}^2+m^2+\eta\right)}$$

that is $\frac{f_c}{L} = \frac{(\mu+1)^2}{\mu}\cdot\frac{2\eta}{(1+\eta)^2}$ which result is the same as (26) and demonstrates that the approximation leading to the common formula for the long wave buckling stress is equivalent to accepting the simple concept of shear deformation enunciated earlier in this section.

/ 7. Long ....

7. LONG, FLAT PANEL WITH CLAMPED EDGES UNDER UNIFORM COMPRESSION

We will define clamping of the edges by the boundary condition $w = \frac{\partial w}{\partial y} = o$ holding thoughout the thickness of the plate. This implies that the filling as well as the faces is fully constrained, a condition not likely to be attained in practice, but it is of interest to investigate the effect of maximum constraint.

Consider long wave buckling, for which the approach outlined in the previous section is suitable.

Equation (40) states:- $f\left(\nabla^2 - \frac{\pi^2 n}{b^2}\right)\frac{\partial^2 w}{\partial x^2} = \frac{L(\mu + 1)^2}{2\mu}\nabla^4 w$

Assume that $w = W(y)\sin\frac{\lambda\pi x}{b}$, which, when substituted into the above equation, gives:-

$$\frac{d^4 W}{dy^4} - \frac{\lambda^2\pi^2}{b^2}(2 - \phi)\frac{d^2 W}{dy^2} + \frac{\lambda^4\pi^4}{b^4}\left[1 - \phi\left(1 + \frac{n}{\lambda^2}\right)\right] = 0 \tag{41}$$

$$\text{where } \phi = \frac{2\mu f}{(\mu + 1)^2 L}$$

The failing stress will be least for the simplest waveform across the panel, so that W must be an even function of y, if the edges are taken to be at $y = \pm b/2$. Therefore put

$$W = A\,\mathrm{ch}\,\frac{2\alpha y}{b} + B\cos\frac{2\beta y}{b}$$

The boundary conditions $w = \frac{\partial w}{\partial y} = 0$ at $y = \pm b/2$ reduce to:-

$$\left.\begin{aligned} A\,\mathrm{ch}\,\alpha + B\cos\beta &= 0 \\ \alpha A\,\mathrm{sh}\,\alpha - \beta B\sin\beta &= 0 \end{aligned}\right\}$$

Hence

$$\left.\begin{aligned} &\alpha\tanh\alpha + \beta\tan\beta = 0 \\ \text{Also}\quad &\alpha^2 - \beta^2 = \frac{\lambda^2\pi^2}{4}(2 - \phi) \\ \text{and}\quad &\alpha^2\beta^2 = \frac{\lambda^4\pi^4}{16}\left[\phi\left(1 + \frac{n}{\lambda^2}\right) - 1\right] \end{aligned}\right\} \tag{42}$$

since W satisfies equation (41)

/ These last ....

These last three equations are solved numerically by finding the least value of η corresponding to a given value of ϕ. It is found that the following expression fits the numerical results within ½%:-

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{\mu} \cdot \frac{3.49\eta^2}{\eta^3 + 3.68\eta^2 + 3.6\eta - 1.3} \tag{43}$$

alternately $f_c = \dfrac{\eta^3}{\eta^3 + 3.68\eta^2 + 3.6\eta - 1.3} \cdot f_e$

where f_e is the equivalent plate critical stress for the clamped edge condition.

Equations (42) break down when η passes through unity with the wavelength tending to zero in a similar fashion to the approximation in the simply supported case.

It will be noted that the critical stresses for both cases tend to the common value of $\frac{L(\mu + 1)^2}{2\mu}$ as $\eta \rightarrow 1$, presumably due to the lessening influence of boundary conditions with decreasing wavelength.

The correction factors to the equivalent plate stresses for both cases are plotted at Fig. 8 and the buckling half wavelengths are plotted against η at Fig. 9.

8. SIMPLY SUPPORTED CURVED PLATES UNDER UNIFORM COMPRESSION

In the first instance we shall regard the problem of curved plates under long wave buckling as an extension of that of flat plates as treated in section 5. We will assume that w and δ can be expanded in powers of h/R, viz:-

$$w = w_o + \frac{hw^{*}}{R} + \frac{h^2 w^{**}}{R^2} + \text{---}$$

$$\delta = \delta_o + \frac{h\delta^{*}}{R} + \frac{h^2\delta^{**}}{R^2} + \text{---}$$

where w_o, w^{*}, w^{**} may be expected to be of the same order. Clearly w_o, δ_o will be the displacements of hypothetical flat plate of the same dimensions constrained to buckle with the same wavelengths (i.e. the same m and n) and are therefore known functions.

Substitute these expansions for w and δ into the equilibrium equations (8), (10), (15) and (19) and neglect second order terms. In effect all terms in $1/R^2$ other than the term $Etw_o/(1-\sigma^2)R^2$ arising in equation (19) are ignored. It should be noted that such terms as Fhw^{*}/R^2 that arise are small since Fh is small compared to Et, while all derivatives are small.

Now since w_o and δ_o are a solution to the equations obtained when the terms in $\frac{1}{R}$ are ignored, the equations of equilibrium of the filling (8 and 10) on the substitution of the expansions, reduce to

$$\left(A\nabla^2 + L\frac{\partial^2}{\partial z^2}\right)\delta^{*} + (F + L)\nabla^2\frac{\partial w^{*}}{\partial z} = F_1\left(u_o, v_o, w_o\right) \qquad (44)$$

$$(F + L)\frac{\partial\delta^{*}}{\partial z} + \left(L\nabla^2 + C\frac{\partial^2}{\partial z^2}\right)w^{*} = F_2\left(u_o, v_o, w_o\right) \qquad (45)$$

where F_1, F_2 are known functions of z independent of w^{*} and δ^{*}

Put $w^{*} = w_1 + w_2 + w_3$

$\delta^{*} = \delta_1 + \delta_2 + \delta_3$

where w_1 and δ_1 form the particular solution of equations 44 and 45 and w_2 and w_3 are the odd and even parts of the complementary solution; δ_2 and δ_3 correspond to w_2 and w_3 and are even and odd functions of z respectively.

/ First suppose ....

First suppose that w_o is an even function of z and therefore δ_o an odd function. From equations 8 & 10 it will be seen that F_1 and F_2 are even and odd respectively, so that w_1 and δ_1 are odd and even respectively, the opposite of w_o and δ_o.

In a similar fashion equation (15) for the equilibrium of the faces reduces to:-

$$\left(\kappa\nabla^2 + \tfrac{1}{2}t\nabla^4\right)w^* - \left(\pm\nabla^2 - \kappa\frac{\partial}{\partial z}\right)\delta^* = F_3\left(u_o, v_o, w_o\right) \qquad (46)$$

where F_3 is an odd function of z, and the top and bottom signs refer to the faces $z = \pm h$

Adding the two forms of this equation corresponding to the two faces together we obtain:-

$$\left(\kappa\nabla^2 + \tfrac{1}{2}t\nabla^4\right)w_3 - \left(\nabla^2 - \kappa\frac{\partial}{\partial z}\right)\delta_3 = 0$$

Now w_o and δ_o are also a solution of this equation and in addition w_o, δ_o and w_3, δ_3 are both complementary solutions of equations (44) and (45) and both w_o and w_3 are even functions and δ_o and δ_3 odd functions. Since both pairs of functions only contain two constants of integration apiece, they must therefore be identical except for a common factor. Hence the addition of $\frac{hw_3}{R}$ to w_o makes no difference to the solution and we may put $w_3 = \delta_3 = 0$.

Consider the equation of equilibrium (19) for the normal forces acting on the faces. Since w_o, δ_o are a solution to the left hand side of this equation taken alone, when $f = f_o$ the failing stress of the hypothetical flat plate, this equation now reduces to

$$\left(f - f_o\right)t\,\frac{\partial^2 w_o}{\partial x^2} + \frac{h}{R}\left\{ft\,\frac{\partial^2 w^*}{\partial x^2} + D\nabla^4 w^* - \tfrac{1}{2}tL\left(\nabla^2 w^* + \frac{\partial\delta^*}{\partial z}\right) \pm C\frac{\partial w^*}{\partial z} \pm F\delta^*\right\}$$
$$= F_4\left(u_o, v_o, w_o\right) - \frac{Etw_o}{(1-\sigma^2)R^2} \qquad (47)$$

where F_4 is an odd function of z.

/ As we ....

As we have shown w^{*} to be an odd function of z and δ^{*} an even function, on adding the two forms of the last equation corresponding to the faces, we obtain:-

$$\left(f - f_o\right) \frac{\partial^2 w_o}{\partial x^2} = - \frac{E w_o}{(1 - \sigma^2) R^2}$$

i.e. $$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \left\{ \frac{\left(\bar{n}^2 + m^2\right)^2}{\bar{n}^2\left(\bar{n}^2 + m^2 + \eta\right)} + \frac{\rho^2}{\eta \bar{n}^2} \right\} \tag{48}$$

where $\rho = 2b^2/\pi^2 R(2h + t)$

For a panel m = 1, and $\bar{n}^2$ has the nearest value to $\dfrac{\eta + (\eta + 1)\rho + \eta\sqrt{\eta\left[\eta + (\eta + 1)\rho^2\right]}}{\eta^2 - \eta - \rho^2}$ provided $\eta > \frac{1}{2}\left[1 + \sqrt{1 + 4\rho^2}\right]$

for η less than this value f has no minimum, so that the approximation fails. The ratio of this stress to that of the corresponding flat panel is plotted at Fig. 11.

The odd functions w_1 and w_2 are not zero; they are determined by the two equations that are obtained by subtracting the forms of equations (46) and (47) corresponding to opposite faces from one another. Hence for curved plates true in phase buckling does not occur.

It has already been observed that in deriving equation (45) for the equilibrium of filling a term of order $\frac{w}{R^2}$ has been ignored, which may in fact be of consequence if the wave length and radius are of the same order. The presence of such a term would make F_2 in equation (45) no longer an odd function and invalidate the argument. Nevertheless, let us extend our results to cover a complete cylinder.

For the complete cylinder $b = \pi R$ and m can take the value zero in equation (48), so:-

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \left\{ \frac{\bar{n}^2}{\bar{n}^2 + \eta} + \frac{\rho^2}{\eta \bar{n}^2} \right\} \tag{49}$$

where $\bar{n}^2$ has nearest value to $\rho\eta/(\eta - \rho)$

and now $\eta = \dfrac{(1 - \sigma^2) L R^2}{E\, ht}$

$$\rho = 2R/(2h + t)$$

/ For a ....

For a long cylinder this may be written:-

$$\frac{f_c}{L} = \tau\left\{1 + \mu - \tfrac{1}{2}\mu\tau\right\} \tag{50}$$

$$\text{where } \tau = \frac{E}{(1-\sigma^2)L} \cdot \frac{t}{R}$$

provided $\tau < 1$. This result is plotted at Fig. 12.

It can be shown that the buckling half wavelength is approximately $\pi(1 - \tau)Rh$, which for normal dimensions will be small compared to the radius but sufficiently large compared to the thickness not to invalidate the equations. It therefore seems probable that the expression for the critical stress is valid provided τ is not too large, but as a check the same result will be obtained by a different method.

At section 6, equations were derived for treating the flat panel as a whole; these will now be extended for curved panels. It will be assumed that equations (37) for the stress couples still hold; this can only be true for symmetrical buckling of a cylinder where the displacement v is zero. Allowances must be made for the effect of the circumferential stresses due to curvature on the normal forces. Since we are dealing with the whole plate we must add the stresses of the two faces together, and these cancel except those due to the y-wise strain w/R arising from the expansion of the cylinder. Hence equation (39) is modified to:-

$$2ft\frac{\partial^2 w}{\partial x^2} + \frac{2Etw}{(1-\sigma^2)R^2} = (2h + t)\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y}\right)$$

$$\text{so that } \left(\nabla^2 - \frac{\pi^2 \eta}{b^2}\right)\left(f\frac{\partial^2}{\partial x^2} + \frac{E}{(1-\sigma^2)R^2}\right)w = \frac{L(\mu+1)^2}{2\mu}\nabla^4 w \tag{51}$$

Putting $w = W\sin\frac{n\,x}{a}\sin\frac{m\,y}{b}$ gives the previous expression

$$\frac{f}{L} = \frac{(\mu+1)^2}{2\mu\bar{n}^2}\left\{\frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2 + m^2 + \eta} + \frac{\rho^2}{\eta}\right\}$$

so that we arrive at the same formulae for the critical stresses for a panel and for a cylinder, by either method despite the fact they form weaker approximations at opposite ends of the range of curvatures considered.

/ A Cylinder ....

A formal solution of the problem of a cylinder in pure bending will now be given.

The stress at failure will not be uniform round the cylinder but proportional to cos(y/R), so that equation (51) requires to be modified as follows:-

$$\left(\nabla^2 - \frac{\eta}{R^2}\right)\left(f \cos\frac{y}{R}\frac{\partial^2}{\partial x^2} + \frac{E}{(1-\sigma^2)R^2}\right) w = \frac{L(\mu+1)^2}{2\mu}\nabla^4 w$$

Put $w = \cos\frac{\bar{n}x}{R}\sum_m W_m \cos\frac{my}{R}$, so that

$$\sum_m\left\{W_m\left(\bar{n}^2 + m^2 + \eta\right)\bar{n}^2\left(f\cos\frac{y}{R} - f_m\right)\cos\frac{my}{R}\right\} = 0$$

where f_m is the value of f given by equation (51) for $\bar{n}$,

Equating coefficients of $\cos\frac{my}{R}$ to zero gives:-

$$\tfrac{1}{2}f\left(\bar{n}^2 + 1 + \eta\right)W_1 - f_o\left(\bar{n}^2+\eta\right)W_o = 0$$

$$\tfrac{1}{2}f\left\{2\left(\bar{n}^2+\eta\right)W_o + \left(\bar{n}^2 + 4 + \eta\right)W_2\right\} - f_o\left(\bar{n}^2 + 1 + \eta\right)W_1 = 0$$

$$\tfrac{1}{2}f\left\{\left(\bar{n}^2 + \overline{m-1}^2 + \eta\right)W_{m-1} + \left(\bar{n}^2 + \overline{m+1}^2 + \eta\right)W_{m+1}\right\} - f_m\left(\bar{n}^2 + m^2 + \eta\right)W_m = 0$$

for $m \geqslant 2$

By neglecting harmonics of order greater than k the stress is given by the discriminant:-

$$\begin{vmatrix} -1/f & b_o & 0 & 0 & . & . & . & 0 & 0 \\ a_1 & -1/f & b_1 & 0 & . & . & . & 0 & 0 \\ 0 & a_2 & -1/f & b_2 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & . & a_k & -1/f \end{vmatrix} =$$

where $a_m = \dfrac{\mu\,\bar{n}^2\left(\bar{n}^2 + \overline{m-1}^2 + \eta\right)}{L(\mu+1)^2\left\{(\bar{n}^2+m^2)^2 + \frac{\rho^2}{\eta}(\bar{n}^2+m^2+\eta)\right\}}$

$$b_m = \frac{\mu\,\bar{n}^2\left(\bar{n}^2 + \overline{m+1}^2 + \eta\right)}{L(\mu+1)^2\left\{(\bar{n}^2+m^2)^2 + \frac{\rho^2}{\eta}(\bar{n}^2+m^2+\eta)\right\}}$$

/ Since ....

Since $\rho^2/\eta \simeq Et/Lh \simeq 100$, say, it is clear that m has to be at least 20 before the coefficients a_m, b_m decrease appreciably, so that it will be necessary to solve a high order discriminant for f in order to attain any accuracy. In addition it will be necessary to determine the value of $\bar{n}$ to give the least stress by trial and error. The method therefore is somewhat impracticable.

/ 10. Simply ....

10. SIMPLY SUPPORTED FLAT PANEL UNDER AN UNIFORM SHEAR LOAD

(a) Short Panels

For short panels in shear energy methods will be used employing the approximations developed in section 6.

Putting $w = \sum_{m,n} W_{mn} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$, we have from equations (38)

$$Z_x = \frac{2h+t}{2h} L\sum\left\{\frac{\bar{n}^2+m^2}{\bar{n}^2+m^2+\eta} \cdot \frac{n\pi}{a} W_{mn} \cos\frac{n\pi x}{a} \sin\frac{m\pi y}{b}\right\}$$

$$Z_y = \frac{2h+t}{2h} L\sum\left\{\frac{\bar{n}^2+m^2}{\bar{n}^2+m^2+\eta} \cdot \frac{m\pi}{b} W_{mn} \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b}\right\}$$

Clearly the boundary conditions are met for edges at $x = o, a$ and $y = o, b$

From equations (36) therefore:-

$$u = \frac{\pi}{b}(h + \tfrac{1}{2}t)\sum\left\{\frac{\bar{n}\, W_{mn}}{\bar{n}^2+m^2+\eta} \cos\frac{n\pi x}{a} \sin\frac{m\pi y}{b}\right\}$$

$$v = \frac{\pi m}{b}(h + \tfrac{1}{2}t)\sum\left\{\frac{m\, W_{mn}}{\bar{n}^2+m^2+\eta} \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b}\right\}$$

Neglecting the contribution from the bending of the faces, the strain energy of the panel is:-

$$\int_o^a\int_o^b\left\{\frac{Et}{1-\sigma^2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + 2\sigma\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1-\sigma}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right] + \frac{h}{L}\left[Z_x^2 + Z_y^2\right]\right\}dxdy$$

$$= \frac{ab}{4}\left\{\frac{Et}{1-\sigma^2}\cdot\frac{\pi^4}{b^4}(h+\tfrac{1}{2}t)^2\eta^2\sum\frac{\left(\bar{n}^2+m^2\right)^2 W_{mn}^2}{\left(\bar{n}^2+m^2+\eta\right)^2} + \frac{L}{h}\frac{\pi^2}{b^2}(h+\tfrac{1}{2}t)^2\sum\frac{\left(\bar{n}^2+m^2\right)^3 W^2_{mn}}{\left(\bar{n}^2+m^2+\eta\right)^2}\right\}$$

$$= \frac{\pi^2}{8}\cdot\frac{at}{b}\cdot\frac{(\mu+1)^2}{\mu} L\sum\frac{\left(\bar{n}^2+m^2\right)^2 W^2_{mn}}{\bar{n}^2+m^2+\eta}$$

/ The work ....

The work done by the shear force 2qt on buckling is:-

$$2qt \int_0^a \int_0^b \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y}\, dxdy$$

$$= 16qt \sum_{\substack{(m-p)\&(n-q)\\ \text{both odd}}} \sum \left\{ \frac{nmpq\; W_{mn}\; W_{pq}}{(m^2 - p^2)(q^2 - n^2)} \right\}$$

Hence $$\frac{q}{L} = \frac{\pi^2}{128} \cdot \frac{a}{b} \cdot \frac{(\mu + 1)^2}{\mu} \quad \frac{\sum \frac{(\bar{n}^2 + m^2)^2 W_{mn}^2}{n^2 + m^2 + \eta}}{\sum\sum \frac{nmpq\; W_{mn} W_{pq}}{(m^2 - p^2)(q^2 - n^2)}}$$

The critical stress is that value of q which makes the following system of equations compatible:-

$$\sum_{p,q} \frac{mnpq(\bar{n}^2 + m^2 + \eta)\; W_{pq}}{(m^2 - p^2)(q^2 - \bar{n}^2)(\bar{n}^2 + m^2)^2} = \lambda W_{mn}, \text{ for all } m,\ n$$

$$\text{where } \lambda = \frac{L}{q} \cdot \frac{\pi^2 a}{64b} \cdot \frac{(\mu + 1)^2}{\mu} \tag{52}$$

If we order the double sequence of terms W_{mn} in some manner so that it can be represented by a single sequence $\Omega_i = W_{mn}$ then the last equation can be written as

$$\sum_{j=1}^{\infty} a_{ij}\,\Omega_j = \lambda\,\Omega_i \quad i = 1 \text{ to } \infty$$

An approximate answer for λ is obtained by considering a system of equations of the form:-

$$\sum_{j=1}^{k} a_{ij}\,\Omega_j = \lambda\,\Omega_i \quad i = 1 \text{ to } k$$

This is tantamount to ignoring all a_{ij}, $i > k$ with $j \leqslant k$, and assuming that λ and Ω_i, $i = 1$ to k, found from the last equation together with $\Omega_i = 0$, $i > k$, is a reasonable approximation. This is unlikely to be the case if any a_{ij}, $i > k$, $j \leqslant k$ are not small though the condition for them to be small is sufficient rather than necessary. When $p = m + 1$, $q = n + 1$ the coefficients of equation (52), on

/ dividing ....

dividing through by η, are approximately $\frac{mn}{4} \cdot \frac{1 + \frac{\bar{n}^2 + m^2}{\eta}}{(\bar{n}^2 + m^2)^2}$ for m and n not small. As m and n increase together this quantity will not be small when $(\bar{n}^2 + m^2)$ is of the order of η Therefore an accurate solution cannot be expected unless η is reasonably large: the minimum value of η will depend on the ratio a/b, for as this departs from one more equations have to be considered since $\bar{n}$ moves more slowly as n increases. For a = b it is suggested that η should be greater than 10.

The equations (52) split into two systems, for since (m - p)&(q - n) are both odd numbers, (p + q) is odd or even whenever (m + n) is odd or even, so that terms W_{ij} with (i + j) both even and odd do not occur in the same equations. The system with (m + n) even may be expected to give the least value of q_c, since it contains terms corresponding to the longest wave length.

If terms involving suffices m and n greater than three are ignored, the characteristic determinant of (52) reduces to:-

$$\begin{vmatrix} \frac{\lambda}{C_{11}} & \frac{4}{9} & 0 & 0 & 0 \\ \frac{4}{9} & \frac{\lambda}{C_{22}} & \frac{4}{5} & -\frac{4}{5} & \frac{36}{25} \\ 0 & \frac{4}{5} & \frac{\lambda}{C_{13}} & 0 & 0 \\ 0 & -\frac{4}{5} & 0 & \frac{\lambda}{C_{31}} & 0 \\ 0 & \frac{36}{25} & 0 & 0 & \frac{\lambda}{C_{33}} \end{vmatrix} = 0$$

i.e. $\lambda^2 = 16C_{22}\left\{\frac{C_{11}}{81} + \frac{C_{13}}{25} + \frac{C_{31}}{25} + \frac{81C_{33}}{625}\right\}$

where $C_{mn} = \frac{\bar{n}^2 + m^2 + \eta}{(\bar{n}^2 + m^2)^2}$

$$= \frac{4(b/a)^2 + 4 + \eta}{50625\left[1 + (b/a)^2\right]^2}\left\{\frac{1354}{1 + (b/a)^2} + \frac{2025}{1 + 9(b/a)^2} + \frac{2025}{9 + (b/a)^2} + \eta\left[\frac{706}{\left(1 + (b/a)^2\right)^2} + \frac{2025}{\left(1 + 9(b/a)^2\right)^2} + \frac{2025}{\left(9 + (b/a)^2\right)^2}\right]\right\}$$

/ Hence for ....

Hence for a square plate;-

$$\frac{q_c}{L} = \frac{4.72(\mu+1)^2}{\mu\sqrt{(\eta+8)(\eta+5.19)}} \tag{5}$$

and for a plate with dimensions in the ration 5 : 4

$$\frac{q_c}{L} = \frac{3.92\ (\mu+1)^2}{\mu\sqrt{(\eta+6.56)(\eta+4.07)}}$$

These results expressed as correction factors to the equivalent plate stress, i.e. $\eta\Big/\sqrt{(\eta+8)(\eta+5.19)}$ and $\eta\Big/\sqrt{(\eta+6.56)(\eta+4.07)}$ are plotted at Fig. 13.

(b) <u>Long Panels</u>

A solution is possible by a method analogous to that used in section 7 for the panel in compression with clamped edg

The counterpart to equation (40) is:-

$$2q\left(\nabla^2 - \frac{\pi^2\eta}{b^2}\right)\frac{\partial^2 w}{\partial x \partial y} = \frac{L(\mu+1)^2}{2\mu}\nabla^4 w$$

$$\begin{aligned}\text{Put } w = {} & \sin\tfrac{\pi}{b}(\lambda x + \alpha y)\left\{A\,\mathrm{ch}\,\tfrac{\pi}{b}\beta y + A'\mathrm{sh}\,\tfrac{\pi}{b}\beta y\right\} \\ & + \cos\tfrac{\pi}{b}(\lambda x + \alpha y)\left\{B\,\mathrm{sh}\,\tfrac{\pi}{b}\beta y + B'\mathrm{ch}\,\tfrac{\pi}{b}\beta y\right\} \\ & + \sin\tfrac{\pi}{b}(\lambda x + \alpha' y)\left\{C\,\mathrm{ch}\,\tfrac{\pi}{b}\beta' y + C'\mathrm{sh}\,\tfrac{\pi}{b}\beta' y\right\} \\ & + \cos\tfrac{\pi}{b}(\lambda x + \alpha' y)\left\{D\,\mathrm{sh}\,\tfrac{\pi}{b}\beta' y + D'\mathrm{ch}\,\tfrac{\pi}{b}\beta' y\right\}\end{aligned} \tag{5}$$

where $\alpha \pm i\beta$ and $\alpha' \pm i\beta'$ are the roots of ν in

$$\nu^4 - 2\phi\lambda\nu^3 + 2\lambda^2\nu^2 - 2\phi\lambda(\lambda^2+\eta)\nu + \lambda^4 = 0 \tag{5}$$

where $\phi = 2\mu q/L(\mu+1)^2$

Take the boundary conditions to be $w = \frac{\partial^2 w}{\partial x^2} = 0$ at $y = \pm b/2$. It is easily shown that either the primed or the un primed constants of integration are zero. The two alternative solutions are the same except for a shift in the origin of x by a quarter wave length.

/The boundary ....

The boundary conditions now reduce to:-

$$\begin{vmatrix} \sec^2 \frac{\pi\alpha}{2} \operatorname{th} \frac{\pi}{2}\beta & \sec^2 \frac{\pi\alpha'}{2} \operatorname{th} \frac{\pi}{2}\beta' & \tan\frac{\pi\alpha}{2} - \tan\frac{\pi\alpha'}{2} \\ 2\alpha\beta\left(1+\tan^2\frac{\pi\alpha}{2}\operatorname{th}^2\frac{\pi}{2}\beta\right) & 2\alpha'\beta'\left(1+\tan^2\frac{\pi\alpha'}{2}\operatorname{th}^2\frac{\pi}{2}\beta'\right) & \left(\gamma^2-\gamma'^2+2\alpha\beta\tan\frac{\pi\alpha}{2}\operatorname{th}\frac{\pi}{2}\beta - 2\alpha'\beta'\tan\frac{\pi\alpha}{2}\operatorname{th}\frac{\pi}{2}\beta'\right) \\ \left(\gamma^2\sec^2\frac{\pi\alpha}{2}\operatorname{th}\frac{\pi}{2}\beta + 2\alpha\beta\tan\frac{\pi\alpha}{2}\operatorname{th}^2\frac{\pi}{2}\beta\right) & \left(\gamma'^2\sec^2\frac{\pi\alpha'}{2}\operatorname{th}\frac{\pi}{2}\beta' + 2\alpha'\beta'\tan\frac{\pi\alpha'}{2}\operatorname{th}\frac{\pi}{2}\beta'\right) & \left(\gamma^2\tan^2\frac{\pi\alpha}{2} - \gamma'^2\tan^2\frac{\pi\alpha'}{2} - 2\alpha\beta\operatorname{th}\frac{\pi}{2}\beta + 2\alpha'\beta'\operatorname{th}\frac{\pi}{2}\beta'\right) \end{vmatrix} = 0 \qquad (57)$$

where $\gamma^2 = \alpha^2 - \beta^2$, $\gamma'^2 = \alpha'^2 - \beta'^2$

From equation (56) we have

$$\alpha + \alpha' = \phi\lambda \qquad (58)$$

$$\alpha^2 + \alpha'^2 + \beta^2 + \beta'^2 + 4\alpha\alpha' = 2\lambda^2 \qquad (59)$$

$$\alpha(\alpha'^2 + \beta'^2) + \alpha'(\alpha^2 + \beta^2) = \phi\lambda(\lambda^2 + \eta) \qquad (60)$$

$$(\alpha^2 + \beta^2)(\alpha'^2 + \beta'^2) = \lambda^4 \qquad (61)$$

The equations could be tackled in the following manner. For a given value of $\phi\lambda$ a set of curves can be plotted of β versus β' for different α by means of equations (57) and (58). A second set of such curves can be plotted by means of equations (58), (59) and (61). The intersection of corresponding curves gives a pair β, β' for every α: these can be converted to a pair λ, η for every α by means of equations (58), (59) and (60). Repeating the process for several values of $\phi\lambda$ a set of curves of λ versus η for different $\phi\lambda$ is obtained this can be converted to a set of curves of ϕ versus λ for different η giving the critical stress for values of η

As the computations involved are longer than can be readily undertaken, resort is made to an approximation analogou to that used by Timoshenko (ref. 3) for the long, homogeneous plate. Assume therefore that

$w = W \sin \frac{\pi y}{b} \sin \frac{\pi}{b}(nx - my)$ where n, m are not necessarily integers.

/ This assumes ....

This assumes that the troughs of the buckling wave form are straight, which is well known not to be so, as is also obvious from equation (55). Furthermore, the conditions of simple support are imperfectly satisfied.

We now proceed as with the short panel and choose m and n to give the least stress. Instead of integrating over the whole panel for the strain energy and work, integrate over the parallelogram $nx - my = 0, b$; $y = 0, b$, and we obtain respectively:-

$$\frac{\pi^2 tL}{4n} \cdot \frac{(\mu + 1)^2}{\mu}\left\{\frac{(n^2 + \overline{1 + m}^2)^2}{n^2 + \overline{1 + m}^2 + \eta} + \frac{(n^2 + \overline{1 - m}^2)^2}{n^2 + \overline{1 - m}^2 + \eta}\right\}$$

and $2m\pi^2 qt$

$$\text{hence } \frac{q}{L} = \frac{(\mu + 1)^2}{8\mu mn}\left\{\frac{(n^2 + \overline{1 + m}^2)^2}{n^2 + \overline{1 + m}^2 + \eta} + \frac{(n^2 + \overline{1 - m}^2)^2}{n^2 + \overline{1 - m}^2 + \eta}\right\}$$

To find m, n to make q least, put $n = 1/r$ and $m = s/r$ and differentiate giving:-

$$\frac{2(r + s)(1 + \overline{r + s}^2)}{1 + \overline{r + s}^2 + \eta r^2} - \frac{(1 + \overline{r + s}^2)^2(\overline{r + s} + \eta r)}{(1 + \overline{r + s}^2 + \eta r^2)^2}$$

$$+ \frac{2(r - s)(1 + \overline{r - s}^2)}{1 + \overline{r - s}^2 + \eta r^2} - \frac{(1 + \overline{r - s}^2)^2(\overline{r - s} + \eta r)}{(1 + \overline{r - s}^2 + \eta r^2)^2} = 0$$

$$\text{and } \frac{4s(r + s)(1 + \overline{r + s}^2)}{1 + \overline{r + s}^2 + \eta r^2} - \frac{2s(r + s)(1 + \overline{r + s}^2)^2}{(1 + \overline{r + s}^2 + \eta r^2)^2} - \frac{(1 + \overline{r + s}^2)^2}{1 + \overline{r + s}^2 + \eta r^2}$$

$$- \frac{4s(r - s)(1 + \overline{r - s}^2)}{1 + \overline{r - s}^2 + \eta r^2} + \frac{2s(r - s)(1 + \overline{r - s}^2)^2}{(1 + \overline{r - s}^2 + \eta r^2)^2} - \frac{(1 + \overline{r - s}^2)^2}{1 + \overline{r - s}^2 + \eta r^2} = 0$$

Approximate by expanding the latter pair of expressions in terms of $1/\eta$ and neglecting second and higher order terms:-

$$(r^2 - s^2 - 1)(r^2 + s^2 + 1) = \frac{1}{\eta r^2}\left\{r^6 - 3r^2(1 + 6s^2 + 5s^4) - 2(1 + s^2)^3\right\}$$

and $s^2(6r^2 + 3s^2 + 2) - (1 + r^2)^2$

$$= \frac{1}{\eta r^2}\left\{s^2(3 + 18r^2 + 9s^2 + 15r^4 + 45r^2s^2 + 5s^4) - (1 + r^2)^3\right\}$$

/ For very large ....

For η very large we find $r = \sqrt{3/2}$ and $s = \sqrt{1/2}$ and as a second approximation we obtain:-

$$r = \sqrt{\frac{3}{2}}\left(1 - \frac{5}{3\eta}\right)$$
$$s = \sqrt{\frac{1}{2}}\left(1 + \frac{1}{\eta}\right)$$

Hence:-

$$\frac{q_c}{L} = \frac{2.83(\mu + 1)^2}{(1 + \eta)}\left\{\frac{0.933\left(1 - \frac{1.09}{\eta} + \frac{0.376}{\eta^2}\right)^2}{1 - \frac{0.177}{\eta} - \frac{0.663}{\eta^2}} + \frac{0.067\left(1 - \frac{2.24}{\eta} + \frac{5.96}{\eta^2}\right)^2}{1 - \frac{2.49}{\eta} + \frac{0.882}{\eta^2}}\right\}$$

i.e. $$\frac{q_c}{L} \simeq \frac{2.83(\mu + 1)^2}{\mu\,\eta}\left\{1 - \frac{3}{\eta} + \frac{5.8}{\eta^2}\right\} \qquad (62)$$

To give the correct stress for very large η the constant in the above equation should be 2.69 instead of 2.83: this discrepancy is the same as that found for the homogeneous panel in ref. 3 and is due to the initial assumptions. It is likely that the other numerical constants in equation (62) will have the same order error.

As η decreases the buckling wave length shortens as in the other cases; also the angle the trough in the wave form makes with the sides sharpens slightly.

At Fig. 13 are plotted the correction factors to the equivalent plate stresses for the three panels for which formulae have been derived.

If a parabolic interpolation of the form $K_1-(K_1- K_2)\left(\frac{b}{a}\right)^2$ is made between the curves in Fig. 13 for the square and the long panel, a good fit is obtained for the curve for a : b = 5 : 4 which suggests that such interpolations may give reasonable results for all values of a : b.

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2.	W.S. Hemp	On a Theory of Sandwich Construction. College of Aeronautics Report No. 15, 1948.
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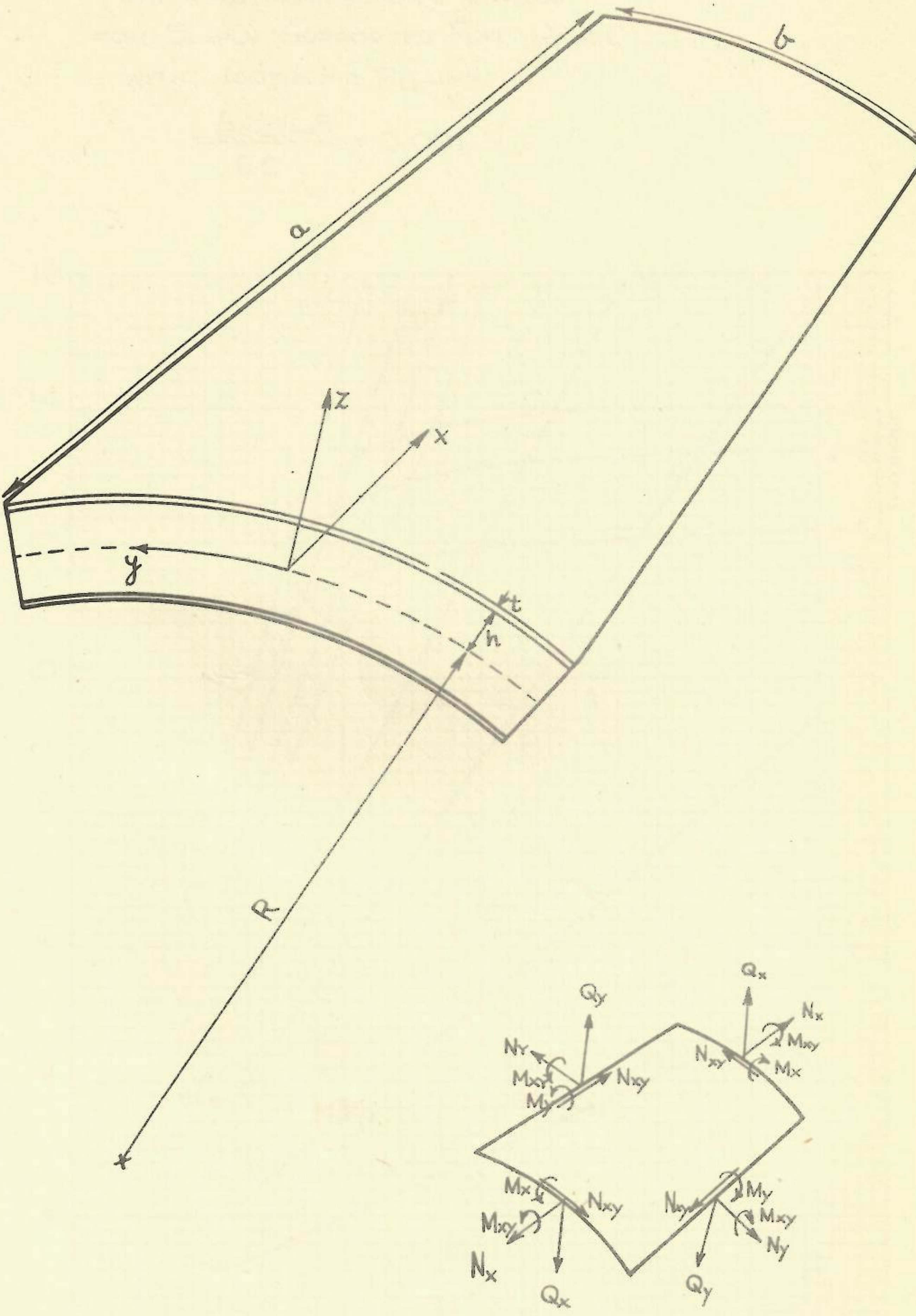


a
b
z
x
y
t
h
R
Q_x
Q_y
N_x
N_y
N_{xy}
M_{xy}
M_x
M_y

CRITICAL COMPRESSIVE STRESS FOR SIMPLY SUPPORTED FLAT PANEL WITH ISOTROPIC FILLING.

$$\frac{L(2c-L)(1-\sigma^2)}{EC} = 0.001$$

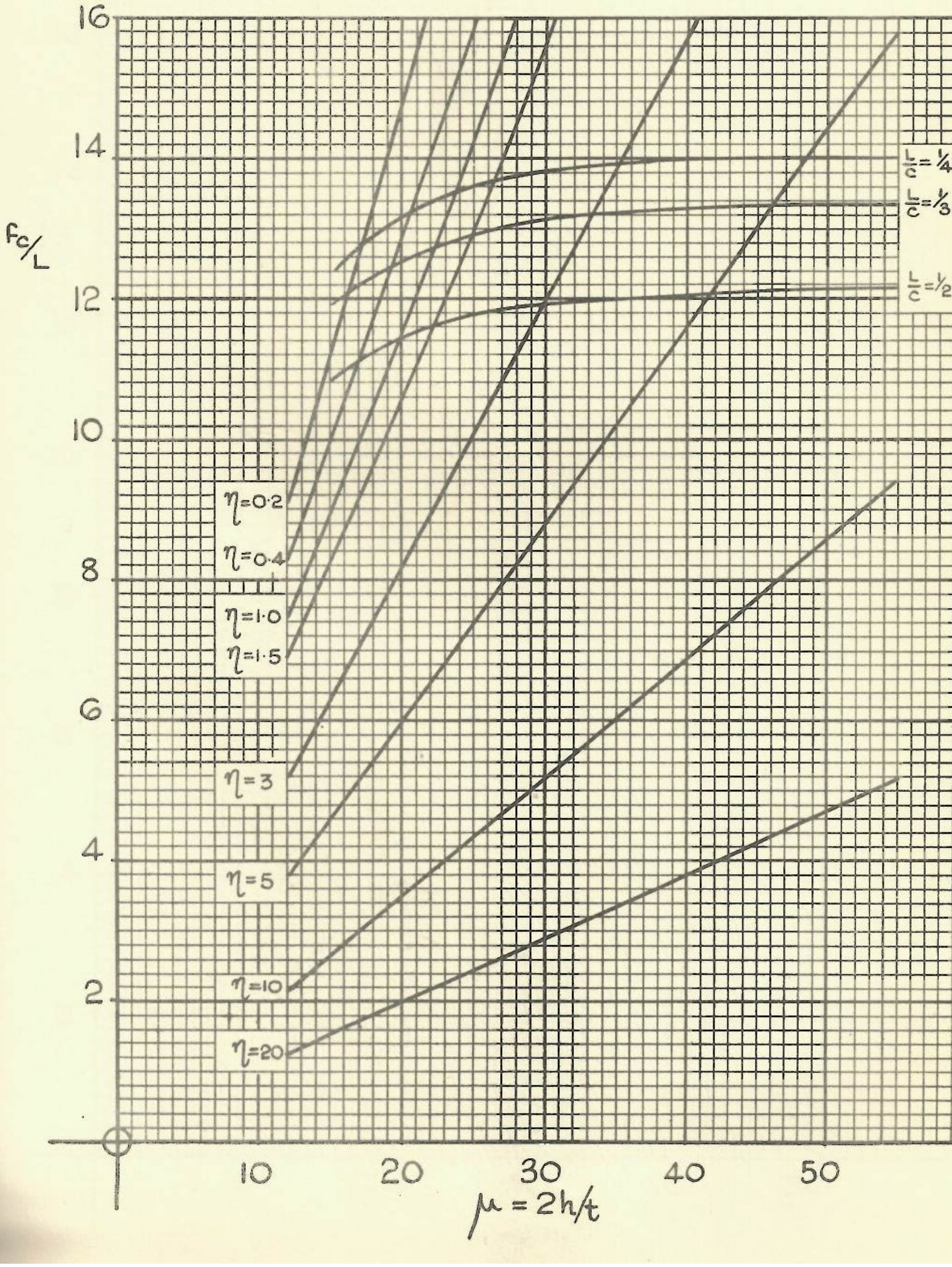


FIG.3.

CRITICAL COMPRESSIVE STRESS FOR SIMPLY SUPPORTED FLAT PANEL WITH ISOTROPIC FILLING $\frac{L(2c-L)(1-\sigma^2)}{Ec} = 0{\cdot}0004$

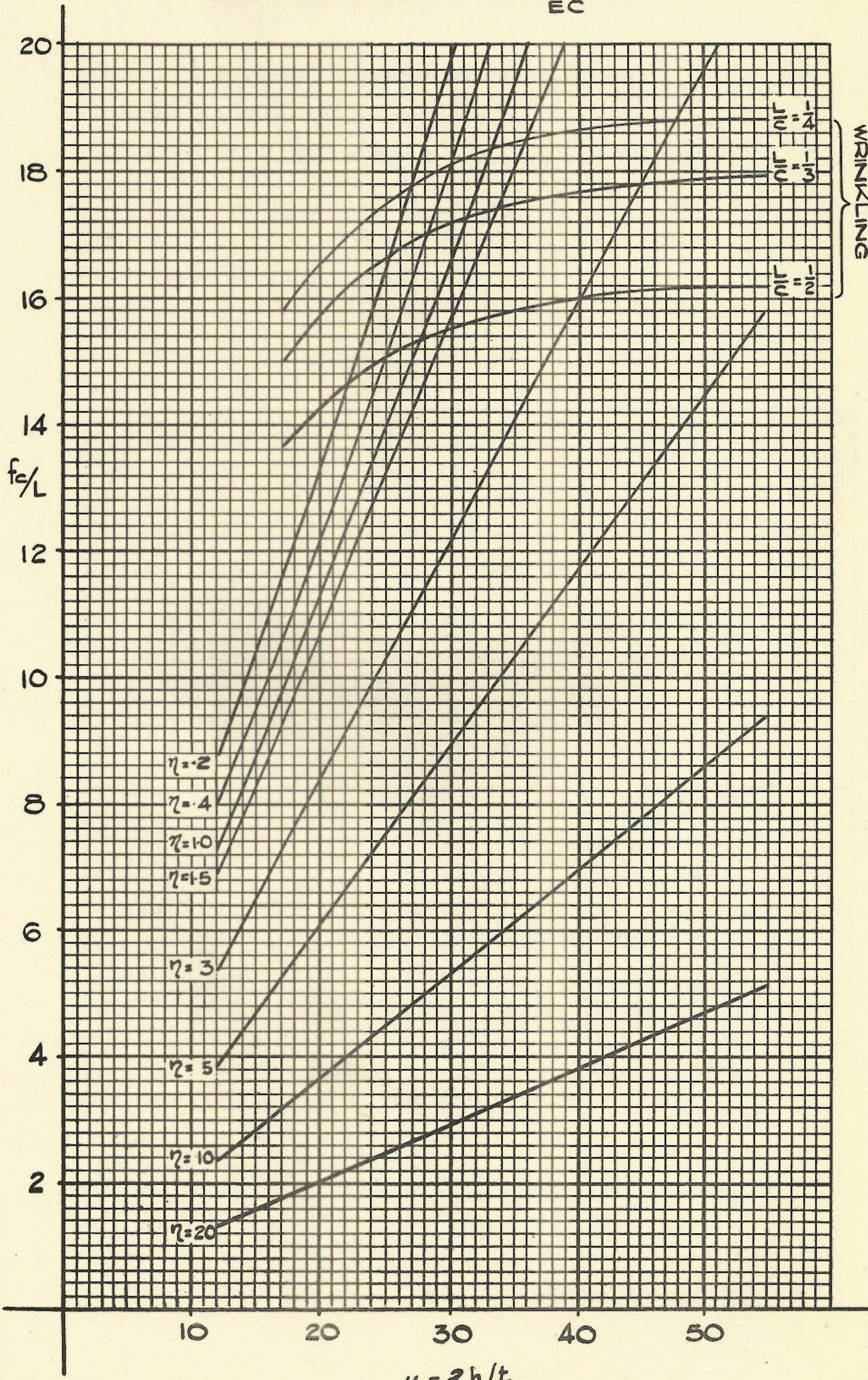


FIG. 4

CRITICAL COMPRESSIVE STRESS

FOR SIMPLY SUPPORTED FLAT PANEL WITH

HONEYCOMB FILLING

$$\frac{L^2(1-\sigma^2)}{EC} = 2{\cdot}5 \times 10^{-4}$$

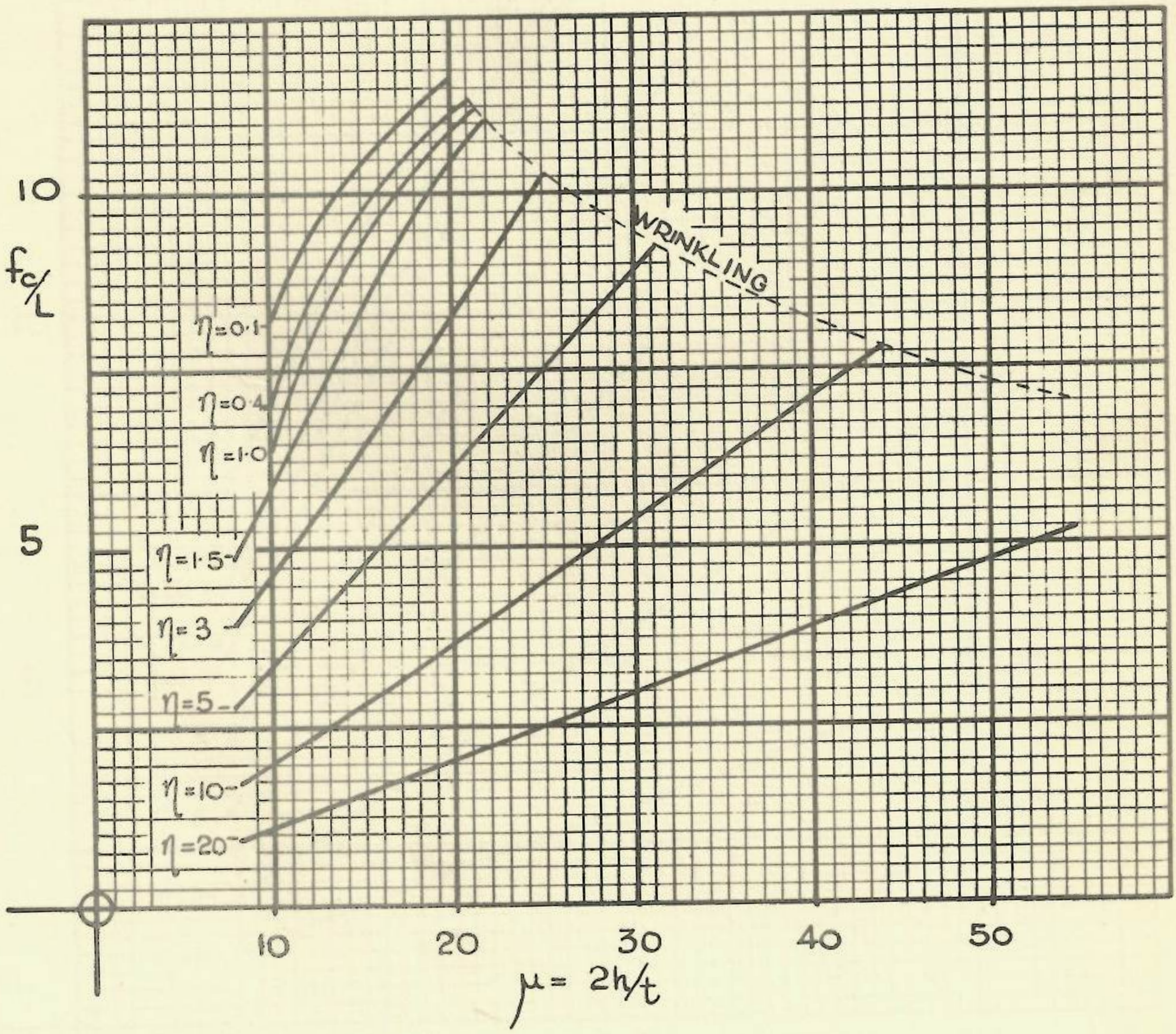
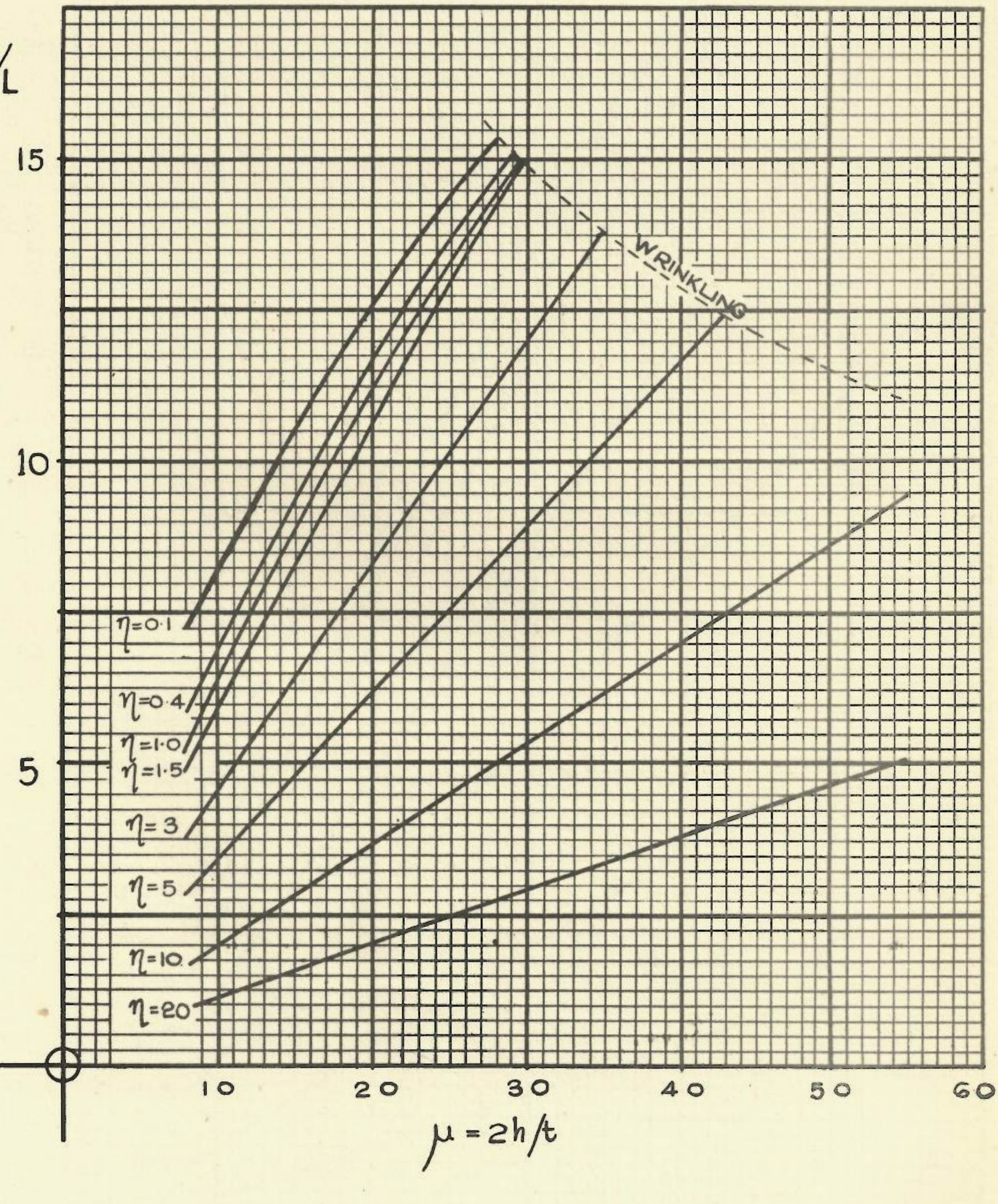


FIG. 5

CRITICAL COMPRESSIVE STRESS
FOR SIMPLY SUPPORTED FLAT PANEL WITH
HONEYCOMB FILLING

$$\frac{L^2(1-\sigma^2)}{EC} = 1{\cdot}0 \times 10^{-4}$$



CRITICAL COMPRESSIVE STRESS
FOR SIMPLY SUPPORTED FLAT PANEL WITH
HONEYCOMB FILLING.

$$\frac{L^2(1-\sigma^2)}{EC} = 2{\cdot}5 \times 10^{-5}$$

FIG. 6.

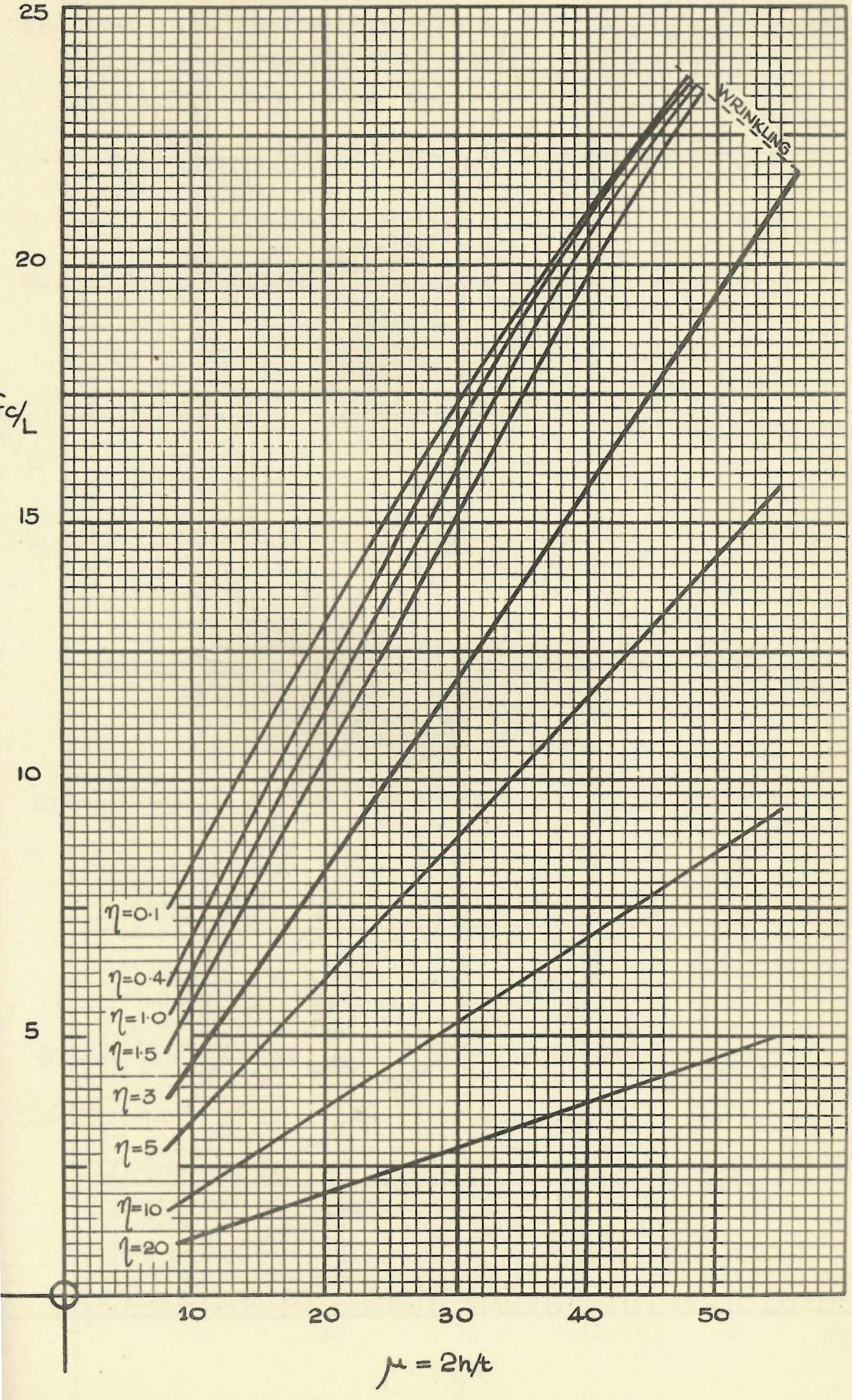
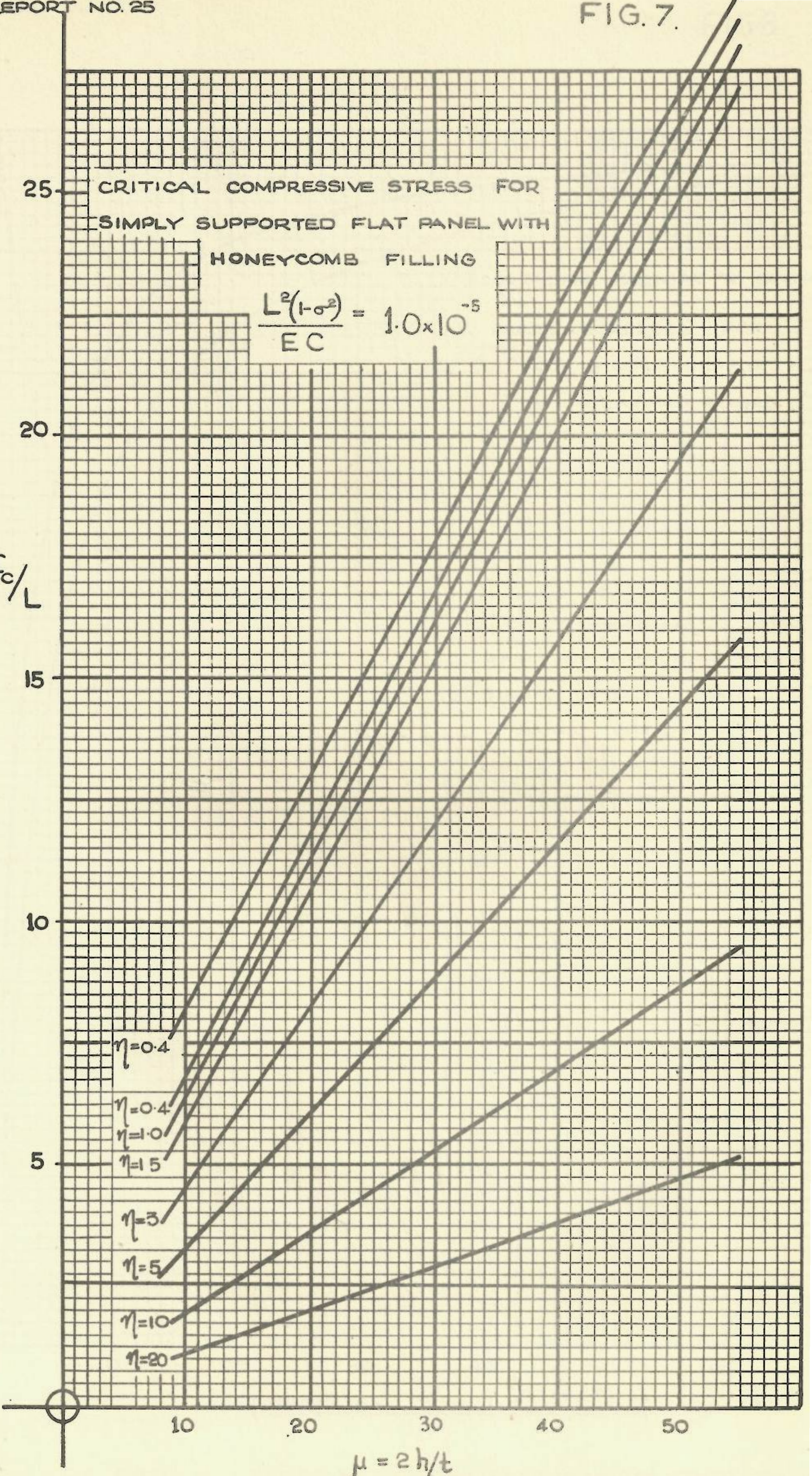
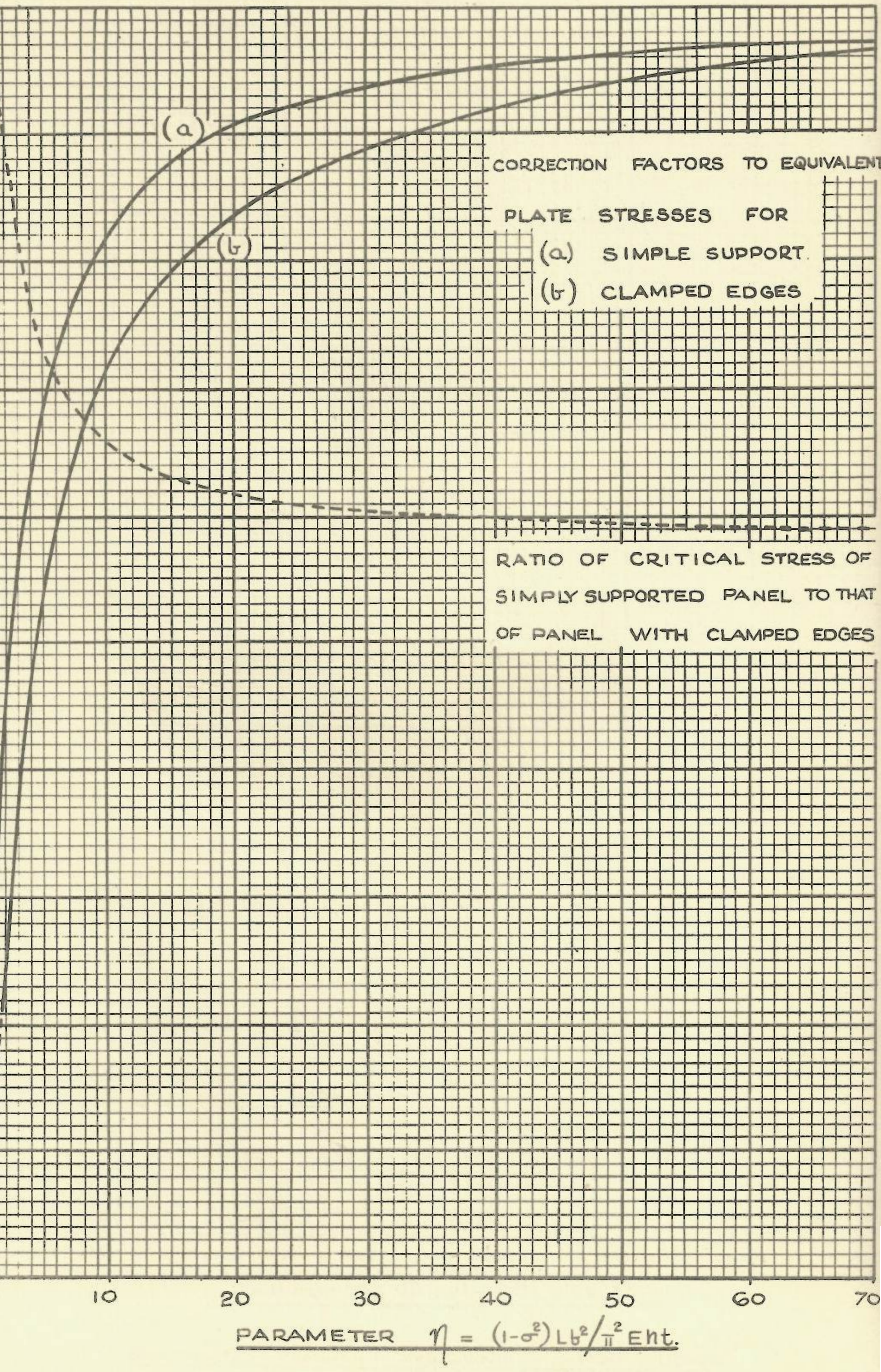


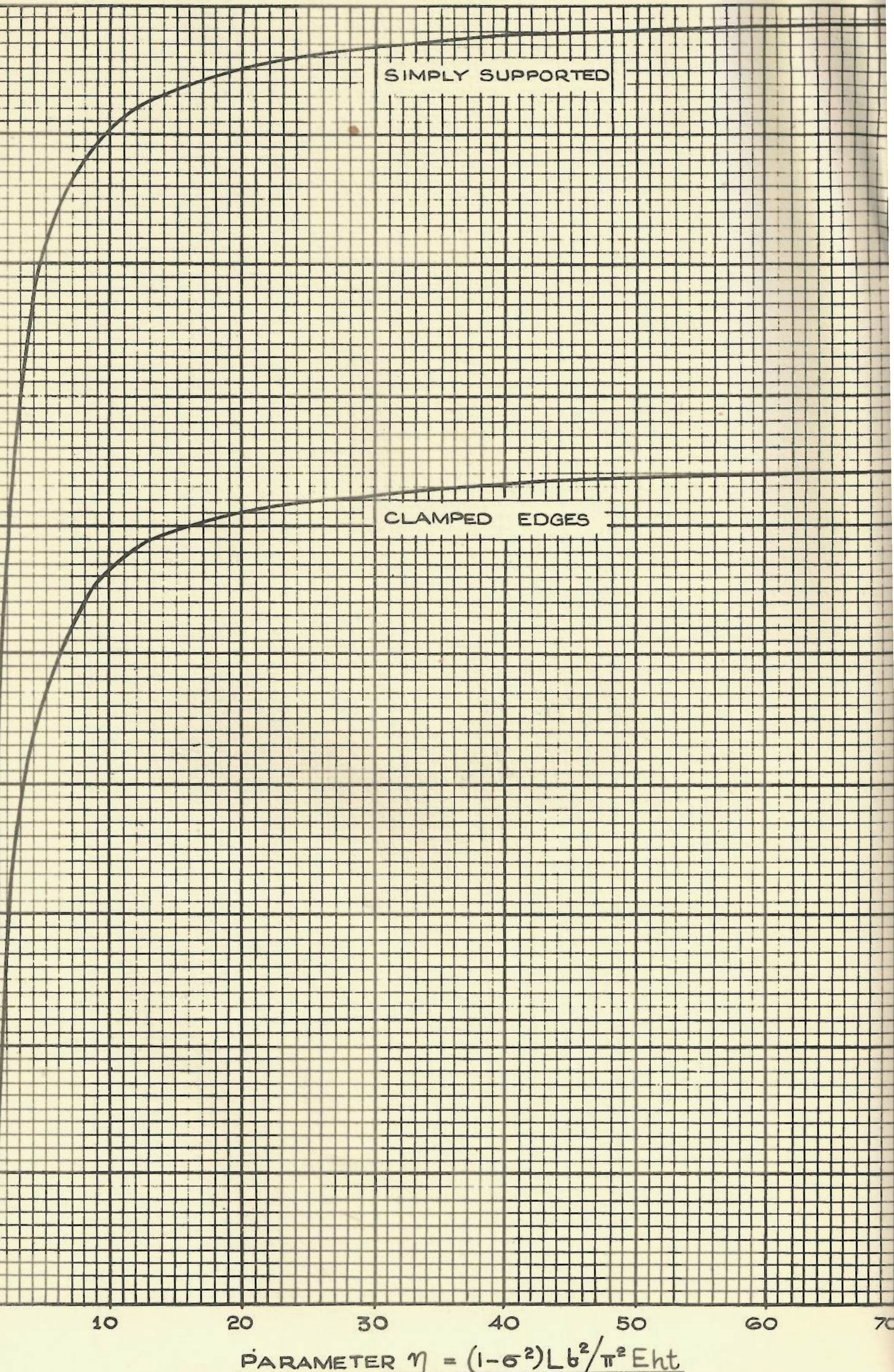
FIG. 7.


CRITICAL COMPRESSIVE STRESS FOR SIMPLY SUPPORTED FLAT PANEL WITH HONEYCOMB FILLING
$\frac{L^2(1-\sigma^2)}{EC} = 1.0\times10^{-5}$
f_c/L
25
20
15
10
5
$\eta=0.4$
$\eta=0.4$
$\eta=1.0$
$\eta=1.5$
$\eta=3$
$\eta=5$
$\eta=10$
$\eta=20$
10
20
30
40
50
$\mu = 2h/t$

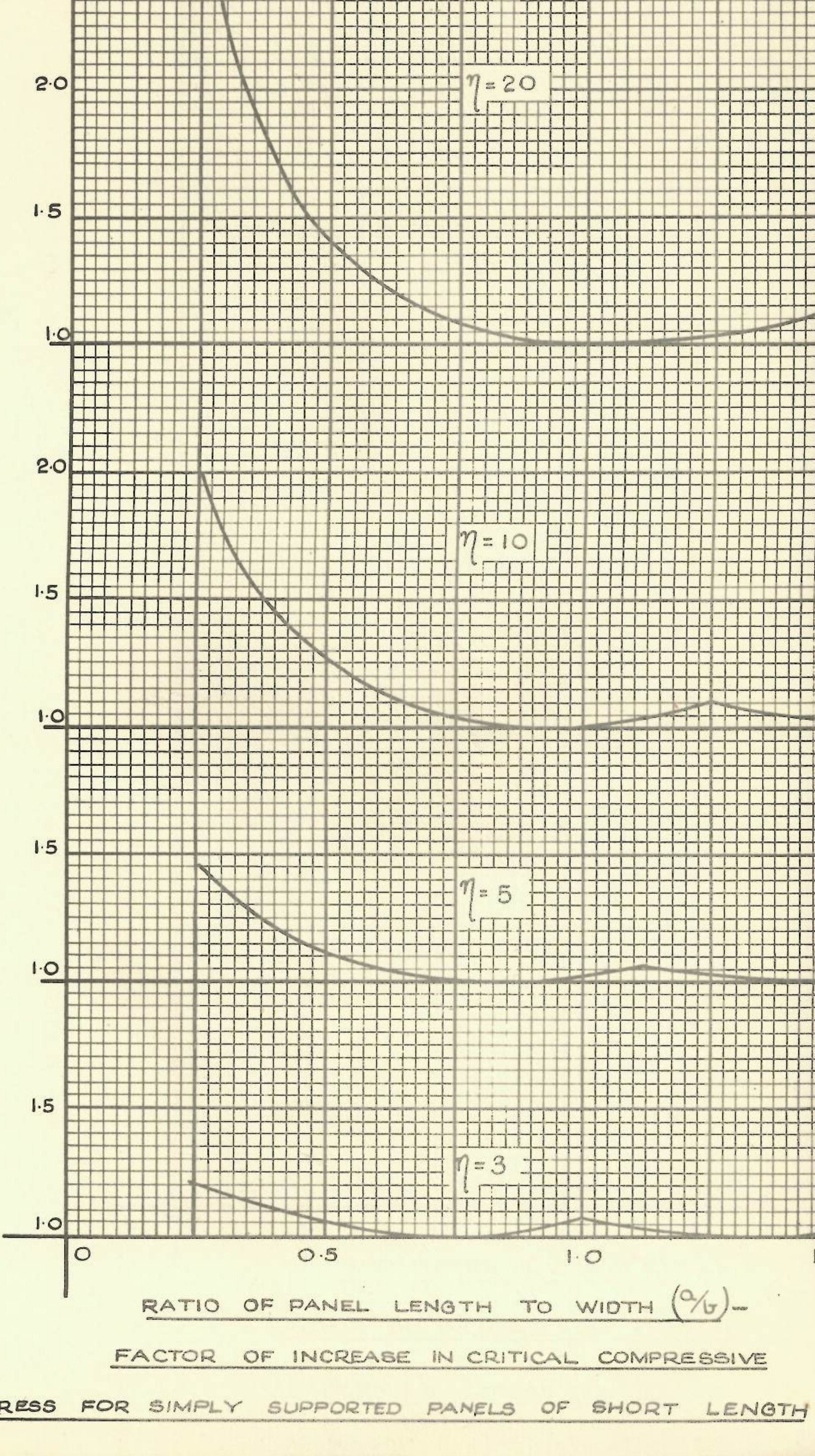


CRITICAL STRESSES OF LONG FLAT PANELS IN COMPRESSION

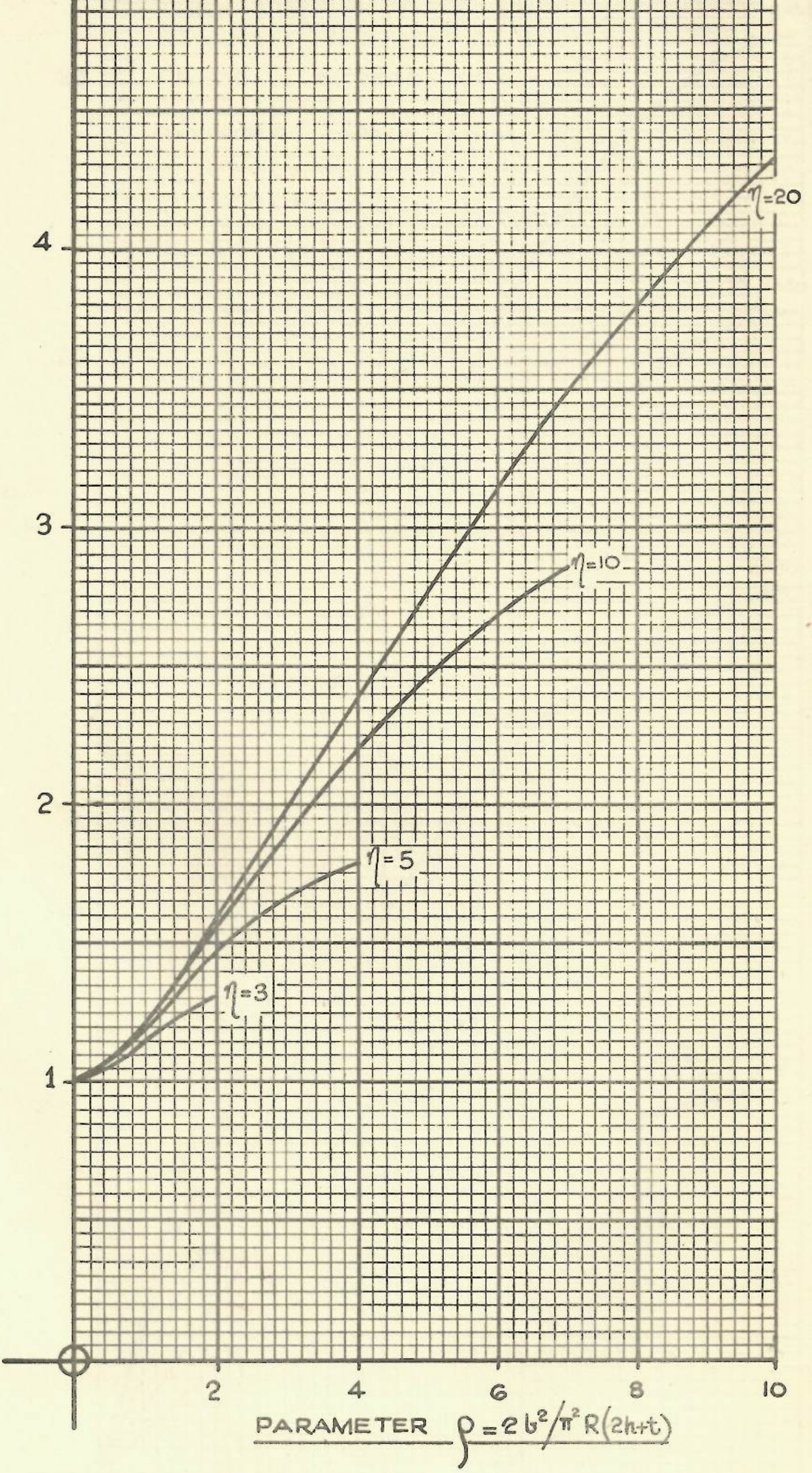
FIG 9



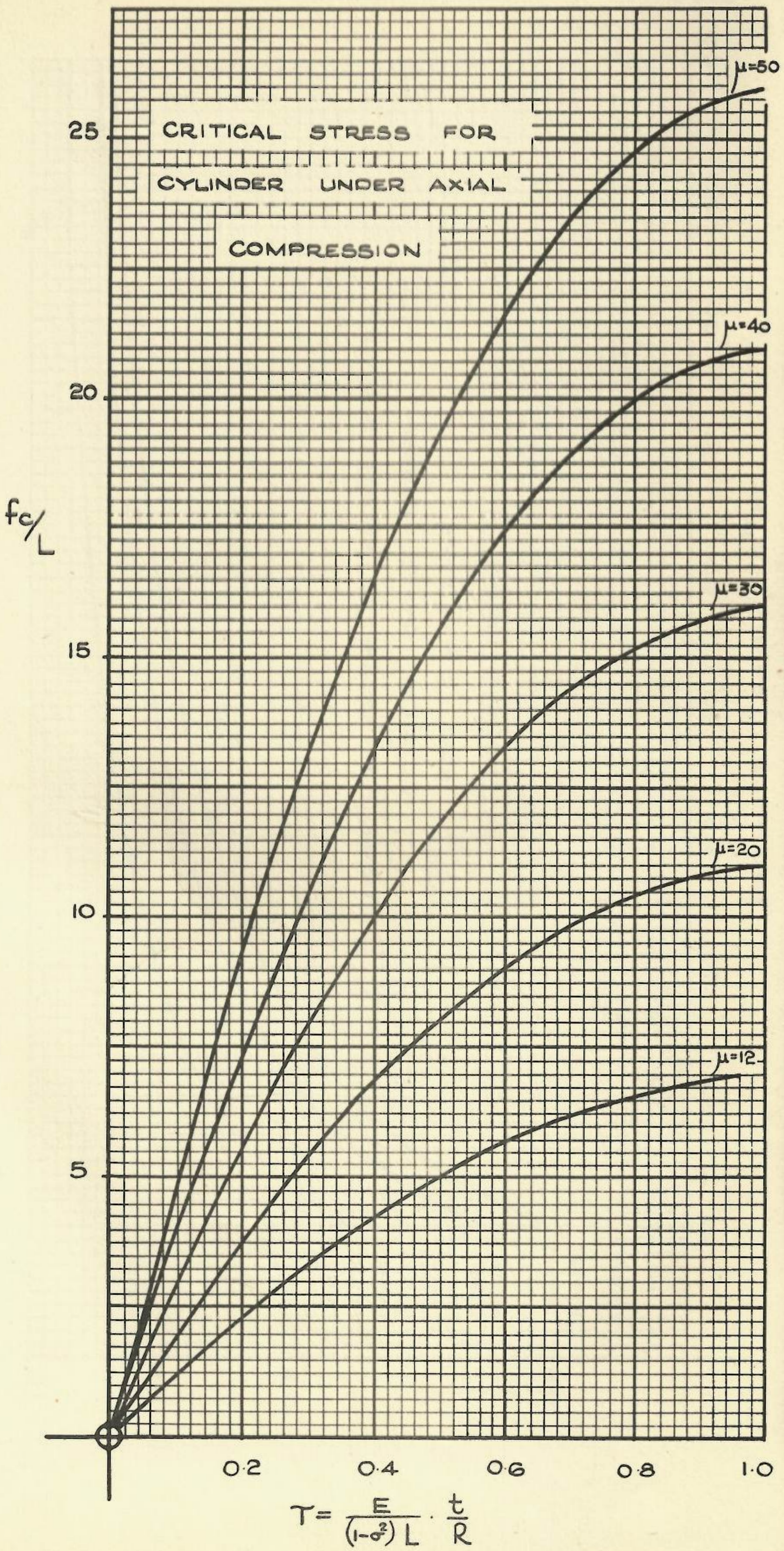
BUCKLING HALF WAVELENGTH FOR LONG FLAT PANELS IN COMPRESSION

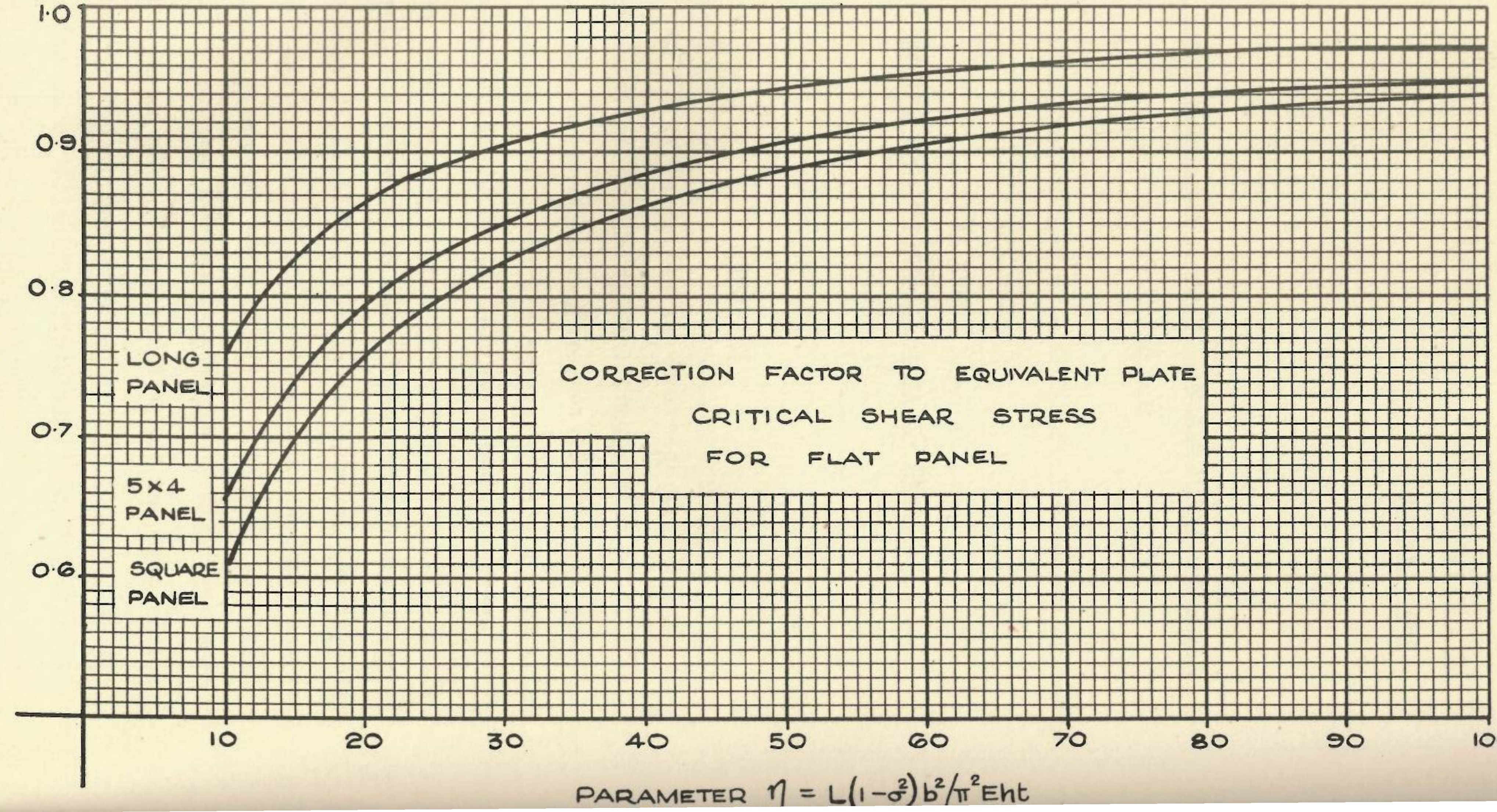


FACTOR OF INCREASE IN CRITICAL COMPRESSIVE STRESS FOR SIMPLY SUPPORTED PANELS OF SHORT LENGTH



FACTOR OF INCREASE IN CRITICAL COMPRESSIVE STRESS DUE TO CURVATURE.


CRITICAL STRESS FOR
CYLINDER UNDER AXIAL
COMPRESSION
25
20
15
10
5
fc/L
μ=50
μ=40
μ=30
μ=20
μ=12
0·2
0·4
0·6
0·8
1·0
$T = \frac{E}{(1-\sigma^2) L} \cdot \frac{t}{R}$



1·0
0·9
0·8
0·7
0·6
LONG PANEL
5×4 PANEL
SQUARE PANEL
CORRECTION FACTOR TO EQUIVALENT PLATE
CRITICAL SHEAR STRESS
FOR FLAT PANEL
10
20
30
40
50
60
70
80
90
PARAMETER $\eta = L(1-\sigma^2)b^2/\pi^2Eht$